

# thm\_2Emeasure\_2EMEASURABLE\_\_UP\_\_LIFT (TMTWEyZ24XxFUHK36NpiieQ3ScegANPVQNd)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2EF$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (1)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (2)$$

**Definition 8** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2E$

**Definition 9** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}}) \quad (3)$$

**Definition 10** We define  $c\_2Epred\_set\_2EPREIMAGE$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V$

**Definition 11** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2E$

Let  $c\_2Emeasure\_2Esubsets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emeasure\_2Esubsets A\_27a \in ( (2^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))}) \quad (4)$$

Let  $c\_2Emeasure\_2Espace : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emeasure\_2Espace A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))}) \quad (5)$$

**Definition 12** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A$ .if  $(\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge P x)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 13** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40$

**Definition 14** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap (c\_2Epred\_set\_2E$

**Definition 15** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap ($

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty ty\_2Enum\_2Enum \quad (6)$$

**Definition 16** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2E\_2E)$ .

**Definition 17** We define  $c\_2Epred\_set\_2EINJ$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^{A\_27a})$

**Definition 18** We define  $c\_2Epred\_set\_2Ecountable$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap (c\_2Ebool\_2E\_3F$

**Definition 19** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_2E) (\lambda V2t \in 2$

**Definition 20** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2E$

**Definition 21** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2E$

**Definition 22** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2E\_2E)$ .

**Definition 23** We define  $c\_2Emeasure\_2Esubset\_class$  to be  $\lambda A\_27a : \iota.\lambda V0sp \in (2^{A\_27a}).\lambda V1sts \in (2^{(2^{A\_27a})})$

**Definition 24** We define  $c\_2Emeasure\_2Ealgebra$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))$

**Definition 25** We define  $c\_2Emeasure\_2Esigma\_algebra$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))$

**Definition 26** We define  $c\_2Epred\_set\_2EFUNSET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0P \in (2^{A\_27a}).\lambda V1Q \in (2^{A\_27a})$

**Definition 27** We define  $c\_2Emeasure\_2Emeasurable$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (8)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (9) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (( \\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (10) \end{aligned}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (11)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\ & p\ V0t)))))) \quad (12) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (13) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in \\ & 2. (((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \quad (14) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (2^{A\_27a}). (\forall V1y \in \\ & (2^{(2^{A\_27a})}). ((ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\ & (2^{A\_27a})\ (2^{(2^{A\_27a})}))\ V0x)\ V1y)) = V0x))) \quad (15) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in (2^{A.27a}). (\forall V1y \in \\ & (2^{(2^{A.27a})}). ((ap\ (c.2Emeasure.2Esubsets\ A.27a)\ (ap\ (ap\ (c.2Epair.2E.2C \\ & (2^{A.27a})\ (2^{(2^{A.27a})})\ V0x)\ V1y)) = V1y))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ & \forall V0a \in (ty.2Epair.2Eprod\ (2^{A.27a})\ (2^{(2^{A.27a})})). (\forall V1b \in \\ & (ty.2Epair.2Eprod\ (2^{A.27b})\ (2^{(2^{A.27b})})). (\forall V2f \in (A.27b^{A.27a}). \\ & ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ (A.27b^{A.27a})\ V2f)\ (ap\ (ap\ (c.2Emeasure.2Emeasurable \\ & A.27a\ A.27b)\ V0a)\ V1b))) \Leftrightarrow ((p\ (ap\ (c.2Emeasure.2Esigma\_algebra \\ & A.27a)\ V0a)) \wedge ((p\ (ap\ (c.2Emeasure.2Esigma\_algebra\ A.27b)\ V1b)) \wedge \\ & ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ (A.27b^{A.27a})\ V2f)\ (ap\ (ap\ (c.2Epred\_set.2EFUNSET \\ & A.27a\ A.27b)\ (ap\ (c.2Emeasure.2Espace\ A.27a)\ V0a))\ (ap\ (c.2Emeasure.2Espace \\ & A.27b)\ V1b)))) \wedge (\forall V3s \in (2^{A.27b}). ((p\ (ap\ (ap\ (c.2Ebool.2EIN \\ & (2^{A.27b})\ V3s)\ (ap\ (c.2Emeasure.2Esubsets\ A.27b)\ V1b))) \Rightarrow (p\ ( \\ & ap\ (ap\ (c.2Ebool.2EIN\ (2^{A.27a})\ (ap\ (ap\ (c.2Epred\_set.2EINTER \\ & A.27a)\ (ap\ (ap\ (c.2Epred\_set.2EPREIMAGE\ A.27a\ A.27b)\ V2f)\ V3s)) \\ & (ap\ (c.2Emeasure.2Espace\ A.27a)\ V0a)))\ (ap\ (c.2Emeasure.2Esubsets \\ & A.27a)\ V0a)))))))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ & \forall V0f \in (A.27b^{A.27a}). (\forall V1P \in (2^{A.27a}). (\forall V2Q \in \\ & (2^{A.27b}). ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ (A.27b^{A.27a})\ V0f)\ (ap\ (ap \\ & (c.2Epred\_set.2EFUNSET\ A.27a\ A.27b)\ V1P)\ V2Q))) \Leftrightarrow (\forall V3x \in \\ & A.27a. ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V3x)\ V1P)) \Rightarrow (p\ (ap\ (ap\ (c.2Ebool.2EIN \\ & A.27b)\ (ap\ V0f\ V3x))\ V2Q)))))) \end{aligned} \quad (18)$$

**Theorem 1**

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ & \forall V0sp \in (2^{A.27a}). (\forall V1a \in (2^{(2^{A.27a})}). (\forall V2b \in \\ & (2^{(2^{A.27a})}). (\forall V3c \in (ty.2Epair.2Eprod\ (2^{A.27b})\ (2^{(2^{A.27b})})). \\ & (\forall V4f \in (A.27b^{A.27a}). ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ (A.27b^{A.27a}) \\ & V4f)\ (ap\ (ap\ (c.2Emeasure.2Emeasurable\ A.27a\ A.27b)\ (ap\ (ap\ (c.2Epair.2E.2C \\ & (2^{A.27a})\ (2^{(2^{A.27a})})\ V0sp)\ V1a))\ V3c))) \wedge ((p\ (ap\ (c.2Emeasure.2Esigma\_algebra \\ & A.27a)\ (ap\ (ap\ (c.2Epair.2E.2C\ (2^{A.27a})\ (2^{(2^{A.27a})})\ V0sp) \\ & V2b))) \wedge (p\ (ap\ (ap\ (c.2Epred\_set.2ESUBSET\ (2^{A.27a})\ V1a)\ V2b)))) \Rightarrow \\ & (p\ (ap\ (ap\ (c.2Ebool.2EIN\ (A.27b^{A.27a})\ V4f)\ (ap\ (ap\ (c.2Emeasure.2Emeasurable \\ & A.27a\ A.27b)\ (ap\ (ap\ (c.2Epair.2E.2C\ (2^{A.27a})\ (2^{(2^{A.27a})})\ V0sp)\ V2b))\ V3c)))))) \end{aligned}$$