

thm_2Emeasure_2EMEASURABLE__UP__SUBSET
(TMcH-
mvd7trVA1h3swyQ7nUyk1o7WR2FhyW9)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Emeasure_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emeasure_2Esubsets A_27a \in ((2^{(2^{A-27a})}) (ty_2Epair_2Eprod (2^{A-27a}) (2^{(2^{A-27a})}))) \tag{2}$$

Definition 7 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A-27a}).(ap V1f V0x)))$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p (ap P x))$ **then** $(the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A-27b})^{A-27a}}) \end{aligned} \quad (3)$$

Definition 11 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A-27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A-27b}}) \end{aligned} \quad (4)$$

Definition 12 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^{A-27a})}). (ap\ (c_2Epred_set_2$

Definition 13 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap\ ($

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (5)$$

Definition 14 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2E2T)$.

Definition 15 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A-27a}). \lambda V1s \in (2^{A-27a})$

Definition 16 We define $c_2Epred_set_2Ecountable$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A-27a}). (ap\ (c_2Ebool_2E_3F$

Definition 17 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2$

Definition 18 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap\ (c_2$

Let $c_2Emeasure_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Espace\ A_27a \in ((2^{A-27a})^{(ty_2Epair_2Eprod\ (2^{A-27a})\ (2^{(2^{A-27a})}))) \quad (6)$$

Definition 19 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap\ (c_2$

Definition 20 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2E2F)$.

Definition 21 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A_27a : \iota. \lambda V0sp \in (2^{A-27a}). \lambda V1sts \in (2^{(2^{A-27a})})$

Definition 22 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A_27a : \iota. \lambda V0a \in (ty_2Epair_2Eprod\ (2^{A-27a})\ (2^{(2^{A-27a})}))$

Definition 23 We define $c_2Emeasure_2Esigma_algebra$ to be $\lambda A_27a : \iota. \lambda V0a \in (ty_2Epair_2Eprod\ (2^{A-27a})\ (2^{(2^{A-27a})}))$

Definition 24 We define $c_2Epred_set_2EPREIMAGE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A-27a}). \lambda V1s \in (2^{A-27a})$

Definition 25 We define $c_2\text{Epred_set_2EINTER}$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Epred_set_2EINTER) A_27a V0s V1t)$

Definition 26 We define $c_2\text{Epred_set_2EFUNSET}$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A_27a}).\lambda V1Q \in (2^{A_27b}).(ap (c_2Epred_set_2EFUNSET) A_27a A_27b V0P V1Q)$

Definition 27 We define $c_2\text{Emeasure_2Emeasurable}$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0a \in (ty_2Epair_2Eprod A_27a A_27b).$

Assume the following.

$$True \tag{7}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t) \Leftrightarrow (p\ V1x))) \Leftrightarrow (p\ V0t)) \tag{8}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \tag{9}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{10}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg \\ & (p\ V0t)))))) \end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in \\ & 2.(((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0sp \in (2^{A_27a}). (\forall V1a \in (2^{(2^{A_27a})}). (\forall V2b \in \\
& \quad (2^{(2^{A_27a})}). (\forall V3c \in (ty_2Epair_2Eprod\ (2^{A_27b})\ (2^{(2^{A_27b})})). \\
& \quad (\forall V4f \in (A_27b^{A_27a}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (A_27b^{A_27a})) \\
V4f)\ (ap\ (ap\ (c_2Emeasure_2E measurable\ A_27a\ A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C \\
(2^{A_27a})\ (2^{(2^{A_27a})})))\ V0sp\ V1a))\ V3c))) \wedge ((p\ (ap\ (c_2Emeasure_2E sigma_algebra \\
A_27a)\ (ap\ (ap\ (c_2Epair_2E_2C\ (2^{A_27a})\ (2^{(2^{A_27a})})))\ V0sp \\
V2b))) \wedge (p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ (2^{A_27a})\ V1a)\ V2b)))) \Rightarrow \\
(p\ (ap\ (ap\ (c_2Ebool_2EIN\ (A_27b^{A_27a}))\ V4f)\ (ap\ (ap\ (c_2Emeasure_2E measurable \\
A_27a\ A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ (2^{A_27a})\ (2^{(2^{A_27a})}))) \\
V0sp\ V2b))\ V3c)))))))))
\end{aligned} \tag{14}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0sp \in (2^{A_27a}). (\forall V1a \in (2^{(2^{A_27a})}). (\forall V2b \in \\
& \quad (2^{(2^{A_27a})}). (\forall V3c \in (ty_2Epair_2Eprod\ (2^{A_27b})\ (2^{(2^{A_27b})})). \\
& \quad ((p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ (2^{A_27a})\ V1a)\ V2b))) \wedge (p \\
& \quad (ap\ (c_2Emeasure_2E sigma_algebra\ A_27a)\ (ap\ (ap\ (c_2Epair_2E_2C \\
(2^{A_27a})\ (2^{(2^{A_27a})})))\ V0sp\ V2b)))) \Rightarrow (p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET \\
(A_27b^{A_27a}))\ (ap\ (ap\ (c_2Emeasure_2E measurable\ A_27a\ A_27b) \\
(ap\ (ap\ (c_2Epair_2E_2C\ (2^{A_27a})\ (2^{(2^{A_27a})})))\ V0sp\ V1a))\ V3c)) \\
(ap\ (ap\ (c_2Emeasure_2E measurable\ A_27a\ A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C \\
(2^{A_27a})\ (2^{(2^{A_27a})})))\ V0sp\ V2b))\ V3c)))))))))
\end{aligned}$$