

# thm\_2Emeasure\_2EMEASURE\_\_PRESERVING\_\_UP\_\_LIFT (TMQJ7NcceKQW5SgX73aatnqfTfD2GLkPHbB)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2EF$

**Definition 7** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A-27a}).(ap V1f V0x)))$

**Definition 8** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (1)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A-27b})^{A-27a}}) \quad (2)$$

**Definition 9** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2E$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A-27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A-27b}}) \quad (3)$$

**Definition 10** We define  $c\_2Epred\_set\_2EPREIMAGE$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (A\_27b^{A\_27a}). \lambda V$

**Definition 11** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap (c\_2E$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (4)$$

Let  $c\_2Emeasure\_2Emeasure : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Emeasure\ A\_27a \in ( (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}) (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})) \quad (5)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (6)$$

**Definition 12** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. c\_2Ebool\_2ET)$ .

**Definition 13** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (A\_27b^{A\_27a}). \lambda V1s \in$

**Definition 14** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A. \mathbf{if} (\exists x \in A. p (ap\ P\ x)) \mathbf{then} (the (\lambda x. x \in A \wedge p (ap\ P\ x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 15** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ V0P (ap (c\_2Emin\_2E\_40$

**Definition 16** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota. \lambda V0P \in (2^{(2^{A\_27a})}). (ap (c\_2Epred\_set\_2E$

**Definition 17** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. \lambda V0f \in (A\_27b^{A\_27c}). \lambda V1$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (7)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (8)$$

**Definition 18** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Ereal\_2Esum : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Esum \in ((ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})}) (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum) \quad (9)$$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (10)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (11)$$

**Definition 19** We define  $c\_Enum\_ESUC$  to be  $\lambda V0m \in ty\_Enum\_Enum.(ap\ c\_Enum\_EABS\_num$

**Definition 20** We define  $c\_Eprim\_rec\_E\_3C$  to be  $\lambda V0m \in ty\_Enum\_Enum.\lambda V1n \in ty\_Enum\_Enum$

**Definition 21** We define  $c\_Earithmic\_E\_3E$  to be  $\lambda V0m \in ty\_Enum\_Enum.\lambda V1n \in ty\_Enum\_Enum$

**Definition 22** We define  $c\_Ebool\_E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_Ebool\_E\_21\ 2)\ (\lambda V2t \in$

**Definition 23** We define  $c\_Earithmic\_E\_3E\_3D$  to be  $\lambda V0m \in ty\_Enum\_Enum.\lambda V1n \in ty\_Enum\_Enum$

Let  $ty\_Ehreal\_Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_Ehreal\_Ehreal \quad (12)$$

Let  $c\_Erealax\_Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_Erealax\_Ereal\_REP\_CLASS \in ((2^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)})^{ty\_Erealax\_Ereal\_REP\_CLASS}) \quad (13)$$

**Definition 24** We define  $c\_Erealax\_Ereal\_REP$  to be  $\lambda V0a \in ty\_Erealax\_Ereal.(ap\ (c\_Emin\_E\_40\ (t$

Let  $c\_Erealax\_Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_Erealax\_Etrealm\_neg \in ((ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)^{ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal}) \quad (14)$$

Let  $c\_Erealax\_Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_Erealax\_Etrealm\_eq \in ((2^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)})^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)}) \quad (15)$$

Let  $c\_Erealax\_Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_Erealax\_Ereal\_ABS\_CLASS \in (ty\_Erealax\_Ereal)^{(2^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)})^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)}} \quad (16)$$

**Definition 25** We define  $c\_Erealax\_Ereal\_ABS$  to be  $\lambda V0r \in (ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)$

**Definition 26** We define  $c\_Erealax\_Ereal\_neg$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal.(ap\ c\_Erealax\_Ereal$

Let  $c\_Erealax\_Etrealm\_add : \iota$  be given. Assume the following.

$$c\_Erealax\_Etrealm\_add \in (((ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)^{ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal})^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)}) \quad (17)$$

**Definition 27** We define  $c\_Erealax\_Ereal\_add$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal.\lambda V1T2 \in ty\_Erealax\_Ereal$

**Definition 28** We define  $c\_Ereal\_Ereal\_sub$  to be  $\lambda V0x \in ty\_Erealax\_Ereal.\lambda V1y \in ty\_Erealax\_Ereal$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (18)$$

Let  $c\_2Erealax\_2Etreal\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_lt \in ((2^{(ty\_2Epair\_2Eprod \ ty\_2Ehreal\_2Ehreal \ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod \ ty\_2Ehreal\_2Ehreal)}) \quad (19)$$

**Definition 29** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

**Definition 30** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$

**Definition 31** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.))$

**Definition 32** We define  $c\_2Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty \ A\_27a \Rightarrow \forall A\_27b.nonempty \ A\_27b \Rightarrow c\_2Epair\_2ESND \\ A\_27a \ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod \ A\_27a \ A\_27b)}) \end{aligned} \quad (20)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty \ A\_27a \Rightarrow \forall A\_27b.nonempty \ A\_27b \Rightarrow c\_2Epair\_2EFST \\ A\_27a \ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod \ A\_27a \ A\_27b)}) \end{aligned} \quad (21)$$

**Definition 33** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27a$

Let  $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty \ A0 \Rightarrow nonempty \ (ty\_2Emetric\_2Emetric \ A0) \quad (22)$$

Let  $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty \ A\_27a \Rightarrow c\_2Emetric\_2Emetric \ A\_27a \in ((ty\_2Emetric\_2Emetric \\ A\_27a)^{(ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod \ A\_27a \ A\_27a)})}) \end{aligned} \quad (23)$$

**Definition 34** We define  $c\_2Emetric\_2Emr1$  to be  $(ap \ (c\_2Emetric\_2Emetric \ ty\_2Erealax\_2Ereal) \ (ap \ (c$

Let  $c\_2Emetric\_2Edist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow c\_2Emetric\_2Edist \ A\_27a \in ((ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod \ A\_27a \ A\_27a)}) \quad (24)$$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty \ A0 \Rightarrow nonempty \ (ty\_2Etopology\_2Etopology \ A0) \quad (25)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty \ A\_27a \Rightarrow c\_2Etopology\_2Etopology \ A\_27a \in \\ ((ty\_2Etopology\_2Etopology \ A\_27a)^{(2^{(2^{A\_27a})})}) \end{aligned} \quad (26)$$

**Definition 35** We define  $c\_Emetric\_EMtop$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Emetric\_2Emetric A\_27a).(ap$   
Let  $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Enets\_2Etends \\ & A\_27a A\_27b \in (((2^{(ty\_2Epair\_2Eprod (ty\_2Etopology\_2Etopology A\_27a) ((2^{A\_27b})^{A\_27b}))})_{A\_27a})_{(A\_27a)^{A\_27b}}) \end{aligned} \quad (27)$$

**Definition 36** We define  $c\_2Eseq\_2E\_2D\_2D\_3E$  to be  $\lambda V0x \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1x$

**Definition 37** We define  $c\_2Eseq\_2Esums$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1s \in ty\_2$

Let  $c\_2Emeasure\_2Emeasurable\_sets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emeasure\_2Emeasurable\_sets \\ & A\_27a \in (((2^{(2^{A\_27a})})_{(ty\_2Epair\_2Eprod (2^{A\_27a}) (ty\_2Epair\_2Eprod (2^{(2^{A\_27a})}) (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))})) \end{aligned} \quad (28)$$

**Definition 38** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

**Definition 39** We define  $c\_2Epred\_set\_2EDISJOINT$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap$

**Definition 40** We define  $c\_2Epred\_set\_2EFUNSET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0P \in (2^{A\_27a}).\lambda V1Q \in (2^{A\_27b}).(ap$

**Definition 41** We define  $c\_2Emeasure\_2Ecountably\_additive$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod$

**Definition 42** We define  $c\_2Emeasure\_2Epositive$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (ty\_2$

Let  $c\_2Emeasure\_2Em\_space : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emeasure\_2Em\_space A\_27a \in \\ & ((2^{A\_27a})_{(ty\_2Epair\_2Eprod (2^{A\_27a}) (ty\_2Epair\_2Eprod (2^{(2^{A\_27a})}) (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))})) \end{aligned} \quad (29)$$

Let  $c\_2Emeasure\_2Esubsets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emeasure\_2Esubsets A\_27a \in ( \\ & (2^{(2^{A\_27a})})_{(ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))} \end{aligned} \quad (30)$$

**Definition 43** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap$

**Definition 44** We define  $c\_2Epred\_set\_2EINJ$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b)^{A\_27a}.\lambda V1s \in (2^{A\_27a}).(ap$

**Definition 45** We define  $c\_2Epred\_set\_2Ecountable$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap (c\_2Ebool\_2E3F$

**Definition 46** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c$

Let  $c\_2Emeasure\_2Espace : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emeasure\_2Espace A\_27a \in ((2^{A\_27a})_{(ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))} \end{aligned} \quad (31)$$

**Definition 47** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2E$

**Definition 48** We define  $c\_2Emeasure\_2Esubset\_class$  to be  $\lambda A\_27a : \iota.\lambda V0sp \in (2^{A\_27a}).\lambda V1sts \in (2^{(2^{A\_27a})})$

**Definition 49** We define  $c\_2Emeasure\_2Ealgebra$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))$

**Definition 50** We define  $c\_2Emeasure\_2Esigma\_algebra$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))$

**Definition 51** We define  $c\_2Emeasure\_2Emeasure\_space$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))$

**Definition 52** We define  $c\_2Emeasure\_2Emeasurable$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))$

**Definition 53** We define  $c\_2Emeasure\_2Emeasure\_preserving$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0m1 \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))$

Assume the following.

$$True \quad (32)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (33)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (34)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (35)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (36)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (37)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (38)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (39)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (40)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{27} \in 2.(\forall V2y \in 2.(\forall V3y_{27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27})))))) \quad (41)$$

Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow (\forall V0P \in (2^{A_{27a}}).(\forall V1a \in A_{27a}.((\exists V2x \in A_{27a}.((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (ap\ V0P\ V1a)))))) \quad (42)$$

Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow (\forall V0sp \in (2^{A_{27a}}).(\forall V1sts \in (2^{(2^{A_{27a}})}).(\forall V2mu \in (ty\_2Erealax\_2Ereal^{(2^{A_{27a}})}).((ap\ (c\_2Emeasure\_2Em\_space\ A_{27a})\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (2^{A_{27a}})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A_{27a}})})\ (ty\_2Erealax\_2Ereal^{(2^{A_{27a}})}))))\ V0sp)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (2^{(2^{A_{27a}})})\ (ty\_2Erealax\_2Ereal^{(2^{A_{27a}})}))\ V1sts)\ V2mu)))) = V0sp)))))) \quad (43)$$

Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow (\forall V0sp \in (2^{A_{27a}}).(\forall V1sts \in (2^{(2^{A_{27a}})}).(\forall V2mu \in (ty\_2Erealax\_2Ereal^{(2^{A_{27a}})}).((ap\ (c\_2Emeasure\_2Emeasurable\_sets\ A_{27a})\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (2^{A_{27a}})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A_{27a}})})\ (ty\_2Erealax\_2Ereal^{(2^{A_{27a}})}))))\ V0sp)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (2^{(2^{A_{27a}})})\ (ty\_2Erealax\_2Ereal^{(2^{A_{27a}})}))\ V1sts)\ V2mu)))) = V1sts)))))) \quad (44)$$

Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow (\forall V0sp \in (2^{A_{27a}}).(\forall V1sts \in (2^{(2^{A_{27a}})}).(\forall V2mu \in (ty\_2Erealax\_2Ereal^{(2^{A_{27a}})}).((ap\ (c\_2Emeasure\_2Emeasure\ A_{27a})\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (2^{A_{27a}})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A_{27a}})})\ (ty\_2Erealax\_2Ereal^{(2^{A_{27a}})}))))\ V0sp)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (2^{(2^{A_{27a}})})\ (ty\_2Erealax\_2Ereal^{(2^{A_{27a}})}))\ V1sts)\ V2mu)))) = V2mu)))))) \quad (45)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0sp \in (2^{A\_27a}). (\forall V1a \in (2^{(2^{A\_27a})}). (\forall V2b \in \\
& \quad (2^{(2^{A\_27a})}). (\forall V3c \in (ty\_2Epair\_2Eprod\ (2^{A\_27b})\ (2^{(2^{A\_27b})}))). \\
& \quad (\forall V4f \in (A\_27b^{A\_27a}). (((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (A\_27b^{A\_27a})) \\
V4f)\ (ap\ (ap\ (c\_2Emeasure\_2E measurable\ A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\
(2^{A\_27a})\ (2^{(2^{A\_27a})})))\ V0sp)\ V1a))\ V3c))) \wedge ((p\ (ap\ (c\_2Emeasure\_2E sigma\_algebra \\
A\_27a)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (2^{A\_27a})\ (2^{(2^{A\_27a})})))\ V0sp) \\
V2b))) \wedge (p\ (ap\ (ap\ (c\_2Epred\_set\_2ESUBSET\ (2^{A\_27a})\ V1a)\ V2b)))))) \Rightarrow \\
& (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (A\_27b^{A\_27a}))\ V4f)\ (ap\ (ap\ (c\_2Emeasure\_2E measurable \\
A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))) \\
V0sp)\ V2b))\ V3c)))))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0x \in A\_27a. (\forall V1y \in A\_27b. (\forall V2a \in A\_27a. (\forall V3b \in \\
& \quad A\_27b. (((ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap\ (ap \\
& \quad (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V2a)\ V3b))) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0f \in ((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}). (\forall V1v \in \\
& \quad A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V1v)\ (ap\ (c\_2Epred\_set\_2EGSPEC \\
& \quad A\_27a\ A\_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A\_27b. ((ap\ (ap\ (c\_2Epair\_2E\_2C \\
& \quad A\_27a\ 2)\ V1v)\ c\_2Ebool\_2ET) = (ap\ V0f\ V2x))))))
\end{aligned} \tag{48}$$



**Theorem 1**

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\
& \quad \forall V0a \in (2^{(2^{A-27a})}). (\forall V1m1 \in (ty\_2Epair\_2Eprod \\
& \quad (2^{A-27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A-27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A-27a})}))). \\
& \quad (\forall V2m2 \in (ty\_2Epair\_2Eprod\ (2^{A-27b})\ (ty\_2Epair\_2Eprod \\
& \quad (2^{(2^{A-27b})})\ (ty\_2Erealax\_2Ereal^{(2^{A-27b})}))). (\forall V3f \in \\
& \quad (A.27b^{A-27a}). ((p\ (ap\ (c\_2Emeasure\_2Emeasure\_space\ A.27a) \\
V1m1))) \wedge ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (A.27b^{A-27a})\ V3f)\ (ap\ (ap\ (c\_2Emeasure\_2Emeasure\_preserving \\
& \quad A.27a\ A.27b)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (2^{A-27a})\ (ty\_2Epair\_2Eprod \\
& \quad (2^{(2^{A-27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A-27a})}))))\ (ap\ (c\_2Emeasure\_2Em\_space \\
& \quad A.27a)\ V1m1))\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (2^{(2^{A-27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A-27a})}))) \\
& \quad V0a)\ (ap\ (c\_2Emeasure\_2Emeasure\ A.27a)\ V1m1))))\ V2m2))) \wedge ((p\ ( \\
& \quad ap\ (c\_2Emeasure\_2Esigma\_algebra\ A.27a)\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\
& \quad (2^{A-27a})\ (2^{(2^{A-27a})}))\ (ap\ (c\_2Emeasure\_2Em\_space\ A.27a) \\
& \quad V1m1))\ (ap\ (c\_2Emeasure\_2Emeasurable\_sets\ A.27a)\ V1m1)))) \wedge \\
& (p\ (ap\ (ap\ (c\_2Epred\_set\_2ESUBSET\ (2^{A-27a})\ V0a)\ (ap\ (c\_2Emeasure\_2Emeasurable\_sets \\
& \quad A.27a)\ V1m1)))))) \Rightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (A.27b^{A-27a})\ V3f) \\
& \quad (ap\ (ap\ (c\_2Emeasure\_2Emeasure\_preserving\ A.27a\ A.27b)\ V1m1) \\
& \quad V2m2))))))
\end{aligned}$$