

thm\_2Emeasure\_2EMEASURE\_REAL\_SUM\_IMAGE  
 (TMXu-  
 AbG83sAZHKk3Ka5o1d3cSQEVRZ76q4n)

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**Definition 1** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \text{ of type } \iota \Rightarrow \iota).$

**Definition 2** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define `c_2Ebool_2ET` to be  $(\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

**Definition 4** We define `c_2Ebool_2EBOUNDED` to be  $(\lambda V0v \in 2. \text{c\_2Ebool\_2ET}).$

**Definition 5** We define `c_2Ecombin_2EC` to be  $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda A. 27c : \iota. (\lambda V0f \in ((A. 27c^{A. 27b})^{A. 27a}))$

**Definition 6** We define `c_2Ebool_2E_21` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A. 27a}). (\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^{A. 27a}))))$

**Definition 7** We define `c_2Ecombin_2Eo` to be  $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda A. 27c : \iota. \lambda V0f \in (A. 27b^{A. 27c}). \lambda V1g$

**Definition 8** We define `c_2Ebool_2EIN` to be  $\lambda A. 27a : \iota. (\lambda V0x \in A. 27a. (\lambda V1f \in (2^{A. 27a}). (\text{ap } V1f \ V0x)))$

**Definition 9** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. \text{inj\_o } (p \Rightarrow q)$  of type  $\iota$ .

**Definition 10** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c\_2Ebool\_2E\_21 } 2) (\lambda V2t \in 2. V2t))))$

**Definition 11** We define `c_2Ebool_2E_3F` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A. 27a}). (\text{ap } V0P \text{ (ap } (\text{c\_2Emin\_2E\_40 } (2^{A. 27a}))))$

Let `ty_2Epair_2Eprod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty\_2Epair\_2Eprod } A0 \ A1) \tag{1}$$

Let `c_2Epair_2EABS_prod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \forall A. 27b. \text{nonempty } A. 27b \Rightarrow \text{c\_2Epair\_2EABS\_prod } A. 27a \ A. 27b \in ((\text{ty\_2Epair\_2Eprod } A. 27a \ A. 27b))^{((2^{A. 27b})^{A. 27a})} \tag{2}$$

**Definition 12** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}})$$
(3)

**Definition 13** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})})$ .

**Definition 14** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a})$ .

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty ty\_2Enum\_2Enum$$
(4)

**Definition 15** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2ET)$ .

**Definition 16** We define  $c\_2Epred\_set\_2EINJ$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^{A\_27a})$ .

**Definition 17** We define  $c\_2Epred\_set\_2Ecountable$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a})$ .

**Definition 18** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^{A\_27a})$ .

Let  $ty\_2Erealx\_2Ereal : \iota$  be given. Assume the following.

$$nonempty ty\_2Erealx\_2Ereal$$
(5)

Let  $c\_2Emeasure\_2Emeasure : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emeasure\_2Emeasure A\_27a \in ( \\ (ty\_2Erealx\_2Ereal^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod (2^{A\_27a}) (ty\_2Epair\_2Eprod (2^{(2^{A\_27a})}) (ty\_2Erealx\_2Ereal^{(2^{A\_27a})})$$
(6)

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega$$
(7)

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega})$$
(8)

**Definition 19** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Ereal\_2Esum : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Esum \in ((ty\_2Erealx\_2Ereal^{(ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum})})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum)$$
(9)

**Definition 20** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 21** We define  $c\_2Ebool\_2E7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E3D\_3D\_3E V0t) c\_2Ebool\_2E21 2))$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (10)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (11)$$

**Definition 22** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num$

**Definition 23** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 24** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 25** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2$

**Definition 26** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (12)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal\_REP\_CLASS}) \quad (13)$$

**Definition 27** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E\_40\ (t$

Let  $c\_2Erealax\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (14)$$

Let  $c\_2Erealax\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (15)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (16)$$

**Definition 28** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 29** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal$

Let  $c\_2Erealax\_2Etrealm\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (17)$$

**Definition 30** We define  $c\_Erealx\_Ereal\_add$  to be  $\lambda V0T1 \in ty\_Erealx\_Ereal.\lambda V1T2 \in ty\_Erealx\_Ereal$

**Definition 31** We define  $c\_Ereal\_Ereal\_sub$  to be  $\lambda V0x \in ty\_Erealx\_Ereal.\lambda V1y \in ty\_Erealx\_Ereal$

Let  $c\_Ereal\_Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_Ereal\_Ereal\_of\_num \in (ty\_Erealx\_Ereal^{ty\_Eenum\_Eenum}) \quad (18)$$

Let  $c\_Erealx\_Etrealt\_lt : \iota$  be given. Assume the following.

$$c\_Erealx\_Etrealt\_lt \in ((2^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)})(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal)) \quad (19)$$

**Definition 32** We define  $c\_Erealx\_Ereal\_lt$  to be  $\lambda V0T1 \in ty\_Erealx\_Ereal.\lambda V1T2 \in ty\_Erealx\_Ereal$

**Definition 33** We define  $c\_Ereal\_Ereal\_lte$  to be  $\lambda V0x \in ty\_Erealx\_Ereal.\lambda V1y \in ty\_Erealx\_Ereal$

**Definition 34** We define  $c\_Ebool\_ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.$

**Definition 35** We define  $c\_Ereal\_Eabs$  to be  $\lambda V0x \in ty\_Erealx\_Ereal.(ap\ (ap\ (ap\ (c\_Ebool\_ECOND$

Let  $c\_Epair\_EESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_Epair\_EESND \\ A\_27a\ A\_27b \in (A\_27b^{(ty\_Epair\_Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (20)$$

Let  $c\_Epair\_EEST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_Epair\_EEST \\ A\_27a\ A\_27b \in (A\_27a^{(ty\_Epair\_Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (21)$$

**Definition 36** We define  $c\_Epair\_EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27a$

Let  $ty\_Emetric\_Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_Emetric\_Emetric\ A0) \quad (22)$$

Let  $c\_Emetric\_Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow c\_Emetric\_Emetric\ A\_27a \in ((ty\_Emetric\_Emetric \\ A\_27a)^{(ty\_Erealx\_Ereal^{(ty\_Epair\_Eprod\ A\_27a\ A\_27a)})}) \end{aligned} \quad (23)$$

**Definition 37** We define  $c\_Emetric\_Emr1$  to be  $(ap\ (c\_Emetric\_Emetric\ ty\_Erealx\_Ereal)\ (ap\ (c\_Emetric\_Emetric$

Let  $c\_Emetric\_Edist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_Emetric\_Edist\ A\_27a \in ((ty\_Erealx\_Ereal^{(ty\_Epair\_Eprod\ A\_27a\ A\_27a)}) \quad (24)$$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \quad (25)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^A-27a)})}) \quad (26)$$

**Definition 38** We define  $c\_2Emetric\_2Emtop$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Emetric\_2Emetric\ A\_27a).(ap$

Let  $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Enets\_2Etends\ A\_27a\ A\_27b \in (((2^{(ty\_2Epair\_2Eprod\ (ty\_2Etopology\_2Etopology\ A\_27a)\ ((2^{A-27b})^{A-27b}))})_{A\_27a})_{(A\_27a^{A-27b})}) \quad (27)$$

**Definition 39** We define  $c\_2Eseq\_2E\_2D\_2D\_3E$  to be  $\lambda V0x \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1x$

**Definition 40** We define  $c\_2Eseq\_2Esums$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1s \in ty\_$

Let  $c\_2Emeasure\_2Emeasurable\_sets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Emeasurable\_sets\ A\_27a \in (((2^{(2^A-27a)})_{(ty\_2Epair\_2Eprod\ (2^{A-27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^A-27a)})\ (ty\_2Erealax\_2Ereal^{(2^A-27a)}))})) \quad (28)$$

**Definition 41** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

**Definition 42** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap\ (c\_$

**Definition 43** We define  $c\_2Epred\_set\_2EDISJOINT$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap\ (c\_$

**Definition 44** We define  $c\_2Epred\_set\_2EFUNSET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0P \in (2^{A-27a}).\lambda V1Q \in ($

**Definition 45** We define  $c\_2Emeasure\_2Ecountably\_additive$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod$

**Definition 46** We define  $c\_2Emeasure\_2Epositive$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A-27a})\ (ty\_$

Let  $c\_2Emeasure\_2Em\_space : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Em\_space\ A\_27a \in (((2^{A-27a})_{(ty\_2Epair\_2Eprod\ (2^{A-27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^A-27a)})\ (ty\_2Erealax\_2Ereal^{(2^A-27a)}))})) \quad (29)$$

Let  $c\_2Emeasure\_2Esubsets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Esubsets\ A\_27a \in ( (2^{(2^A-27a)})_{(ty\_2Epair\_2Eprod\ (2^{A-27a})\ (2^{(2^A-27a)}))}) \quad (30)$$

**Definition 47** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Epred\_set\_2EUNION) s t)$ .  
Let  $c\_2Emeasure\_2Espace : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emeasure\_2Espace A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))}) \quad (31)$$

**Definition 48** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Epred\_set\_2EDIFF) s t)$ .

**Definition 49** We define  $c\_2Emeasure\_2Esubset\_class$  to be  $\lambda A\_27a : \iota.\lambda V0sp \in (2^{A\_27a}).\lambda V1sts \in (2^{(2^{A\_27a})})$ .

**Definition 50** We define  $c\_2Emeasure\_2Ealgebra$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))$ .

**Definition 51** We define  $c\_2Emeasure\_2Esigma\_algebra$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))$ .

**Definition 52** We define  $c\_2Emeasure\_2Emeasure\_space$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))$ .

**Definition 53** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap (c\_2Epred\_set\_2EINSERT) s x)$ .

**Definition 54** We define  $c\_2Epred\_set\_2EDELETE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1x \in A\_27a.(ap (c\_2Epred\_set\_2EDELETE) s x)$ .

**Definition 55** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap (c\_2Ebool\_2E21) s)$ .

Let  $c\_2Epred\_set\_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EITSET A\_27a A\_27b \in (((A\_27b^{A\_27b})^{(2^{A\_27a})})^{((A\_27b^{A\_27b})^{A\_27a})}) \quad (32)$$

**Definition 56** We define  $c\_2Ereal\_sigma\_2EREAL\_SUM\_IMAGE$  to be  $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Erealax\_2E2)$ .

Assume the following.

$$True \quad (33)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (34)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (35)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t) \Leftrightarrow (p V0t)))) \quad (36)$$

Assume the following.

$$(\forall V0t \in 2.((\neg (p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (37)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (38)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (39)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)) \quad (40)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \quad (41)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (42)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (43)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (44)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\exists V1x \in A\_27a.(p(ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A\_27a.(\neg(p(ap V0P V2x)))))) \quad (45)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p(ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A\_27a.(p(ap V1P V3x)) \vee (p V0Q)))))) \quad (46)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B)))))) \quad (47)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B)) \wedge (p V2C)))) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C)))) \quad (48)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V1B) \wedge (p V2C)) \vee (p V0A)))) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))) \quad (49)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))) \quad (50)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (p V2t3)))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))) \quad (51)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_{.27} \in 2. (\forall V2y \in 2. (\forall V3y_{.27} \in 2. (((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (52)$$

Assume the following.

$$\forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0f \in (2^{A_{.27a}}). (\forall V1v \in A_{.27a}. ((\forall V2x \in A_{.27a}. ((V2x = V1v) \Rightarrow (p (ap V0f V2x)))) \Leftrightarrow (p (ap V0f V1v)))))) \quad (53)$$

Assume the following.

$$(\forall V0v \in 2. ((p (ap c_{.2E} \text{bool}_{.2E} \text{BOUNDED } V0v)) \Leftrightarrow \text{True})) \quad (54)$$

Assume the following.

$$\forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0x \in (2^{A_{.27a}}). (\forall V1y \in (2^{(2^{A_{.27a}})}). ((ap (c_{.2E} \text{measure}_{.2E} \text{subsets } A_{.27a}) (ap (ap (c_{.2E} \text{pair}_{.2E} \text{2C } (2^{A_{.27a}}) (2^{(2^{A_{.27a}})})) V0x) V1y)) = V1y)))) \quad (55)$$

Assume the following.

$$\forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0a \in (ty_{.2E} \text{pair}_{.2E} \text{prod } (2^{A_{.27a}}) (2^{(2^{A_{.27a}})})). (\forall V1s \in (2^{A_{.27a}}). (\forall V2t \in (2^{A_{.27a}}). (((p (ap (c_{.2E} \text{measure}_{.2E} \text{algebra } A_{.27a}) V0a)) \wedge ((p (ap (ap (c_{.2E} \text{bool}_{.2E} \text{IN } (2^{A_{.27a}}) V1s) (ap (c_{.2E} \text{measure}_{.2E} \text{subsets } A_{.27a}) V0a))) \wedge (p (ap (ap (c_{.2E} \text{bool}_{.2E} \text{IN } (2^{A_{.27a}}) V2t) (ap (c_{.2E} \text{measure}_{.2E} \text{subsets } A_{.27a}) V0a)))))) \Rightarrow (p (ap (ap (c_{.2E} \text{bool}_{.2E} \text{IN } (2^{A_{.27a}}) (ap (ap (c_{.2E} \text{pred}_{.2E} \text{set}_{.2E} \text{DIFF } A_{.27a}) V1s) V2t)) (ap (c_{.2E} \text{measure}_{.2E} \text{subsets } A_{.27a}) V0a))))))))) \quad (56)$$



Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod \\
& (2^{A-27a}) (ty\_2Epair\_2Eprod (2^{(2^{A-27a})}) (ty\_2Erealax\_2Ereal(2^{A-27a}))))). \\
& ((p (ap (c\_2Emeasure\_2Emeasure\_space\ A.27a)\ V0m)) \Rightarrow ((ap (ap ( \\
& c\_2Emeasure\_2Emeasure\ A.27a)\ V0m) (c\_2Epred\_set\_2EEMPTY\ A.27a)) = \\
& (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0)))) \\
& \tag{57}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod \\
& (2^{A-27a}) (ty\_2Epair\_2Eprod (2^{(2^{A-27a})}) (ty\_2Erealax\_2Ereal(2^{A-27a}))))). \\
& (\forall V1s \in (2^{A-27a}). (\forall V2t \in (2^{A-27a}). (\forall V3u \in \\
& (2^{A-27a}). (((p (ap (c\_2Emeasure\_2Emeasure\_space\ A.27a)\ V0m)) \wedge \\
& ((p (ap (ap (c\_2Ebool\_2EIN (2^{A-27a})\ V1s) (ap (c\_2Emeasure\_2Emeasurable\_sets \\
& A.27a)\ V0m))) \wedge ((p (ap (ap (c\_2Ebool\_2EIN (2^{A-27a})\ V2t) (ap (c\_2Emeasure\_2Emeasurable\_sets \\
& A.27a)\ V0m))) \wedge ((p (ap (ap (c\_2Epred\_set\_2EDISJOINT\ A.27a)\ V1s) \\
& V2t)) \wedge (V3u = (ap (ap (c\_2Epred\_set\_2EUNION\ A.27a)\ V1s)\ V2t)))))) \Rightarrow \\
& ((ap (ap (c\_2Emeasure\_2Emeasure\ A.27a)\ V0m)\ V3u) = (ap (ap\ c\_2Erealax\_2Ereal\_add \\
& (ap (ap (c\_2Emeasure\_2Emeasure\ A.27a)\ V0m)\ V1s) (ap (ap (c\_2Emeasure\_2Emeasure \\
& A.27a)\ V0m)\ V2t))))))))) \\
& \tag{58}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A-27a}). (\forall V1t \in \\
& (2^{A-27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A.27a. ((p (ap (ap (c\_2Ebool\_2EIN \\
& A.27a)\ V2x)\ V0s)) \Leftrightarrow (p (ap (ap (c\_2Ebool\_2EIN\ A.27a)\ V2x)\ V1t)))))) \\
& \tag{59}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A-27a}). (\forall V1t \in \\
& (2^{A-27a}). ((p (ap (ap (c\_2Epred\_set\_2EDISJOINT\ A.27a)\ V0s)\ V1t)) \Leftrightarrow \\
& (\neg (\exists V2x \in A.27a. ((p (ap (ap (c\_2Ebool\_2EIN\ A.27a)\ V2x)\ V0s)) \wedge \\
& (p (ap (ap (c\_2Ebool\_2EIN\ A.27a)\ V2x)\ V1t)))))) \\
& \tag{60}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A-27a}). (\forall V1t \in \\
& (2^{A-27a}). (\forall V2x \in A.27a. ((p (ap (ap (c\_2Ebool\_2EIN\ A.27a) \\
& V2x) (ap (ap (c\_2Epred\_set\_2EDIFF\ A.27a)\ V0s)\ V1t)) \Leftrightarrow ((p (ap ( \\
& ap (c\_2Ebool\_2EIN\ A.27a)\ V2x)\ V0s)) \wedge (\neg (p (ap (ap (c\_2Ebool\_2EIN \\
& A.27a)\ V2x)\ V1t)))))) \\
& \tag{61}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in \\
& A.27a. (\forall V2s \in (2^{A-27a}). ((p (ap (ap (c\_2Ebool\_2EIN\ A.27a) \\
& V0x) (ap (ap (c\_2Epred\_set\_2EINSERT\ A.27a)\ V1y)\ V2s)) \Leftrightarrow ((V0x = \\
& V1y) \vee (p (ap (ap (c\_2Ebool\_2EIN\ A.27a)\ V0x)\ V2s)))))) \\
& \tag{62}
\end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1s \in \\ (2^{A.27a}). ((\neg(p\ (ap\ (ap\ (c.2Ebool\_2EIN\ A.27a)\ V0x)\ V1s))) \Leftrightarrow ((ap \\ (ap\ (c.2Epred\_set\_2EDELETE\ A.27a)\ V1s)\ V0x) = V1s)))) \end{aligned} \quad (63)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in \\ A.27a. ((p\ (ap\ (ap\ (c.2Ebool\_2EIN\ A.27a)\ V0x)\ (ap\ (ap\ (c.2Epred\_set\_2EINSERT \\ A.27a)\ V1y)\ (c.2Epred\_set\_2EEMPTY\ A.27a)))) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (64)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1x \in \\ A.27a. ((ap\ (ap\ (c.2Epred\_set\_2EINSERT\ A.27a)\ V1x)\ V0s) = (ap\ ( \\ ap\ (c.2Epred\_set\_2EUNION\ A.27a)\ (ap\ (ap\ (c.2Epred\_set\_2EINSERT \\ A.27a)\ V1x)\ (c.2Epred\_set\_2EEMPTY\ A.27a)))) V0s)))) \end{aligned} \quad (65)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(2^{A.27a})}). (( \\ (p\ (ap\ V0P\ (c.2Epred\_set\_2EEMPTY\ A.27a))) \wedge (\forall V1s \in (2^{A.27a}). \\ ((p\ (ap\ (c.2Epred\_set\_2EFINITE\ A.27a)\ V1s)) \wedge (p\ (ap\ V0P\ V1s)))) \Rightarrow \\ (\forall V2e \in A.27a. ((\neg(p\ (ap\ (ap\ (c.2Ebool\_2EIN\ A.27a)\ V2e)\ V1s))) \Rightarrow \\ (p\ (ap\ V0P\ (ap\ (ap\ (c.2Epred\_set\_2EINSERT\ A.27a)\ V2e)\ V1s)))))) \Rightarrow \\ (\forall V3s \in (2^{A.27a}). ((p\ (ap\ (c.2Epred\_set\_2EFINITE\ A.27a)\ \\ V3s)) \Rightarrow (p\ (ap\ V0P\ V3s)))))) \end{aligned} \quad (66)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in (ty\_2Erealax\_2Ereal^{A.27a}). \\ (((ap\ (ap\ (c.2Ereal\_sigma\_2EREAL\_SUM\_IMAGE\ A.27a)\ V0f)\ (c.2Epred\_set\_2EEMPTY \\ A.27a)) = (ap\ c.2Ereal\_2Ereal\_of\_num\ c.2Enum\_2E0)) \wedge (\forall V1e \in \\ A.27a. (\forall V2s \in (2^{A.27a}). ((p\ (ap\ (c.2Epred\_set\_2EFINITE \\ A.27a)\ V2s)) \Rightarrow ((ap\ (ap\ (c.2Ereal\_sigma\_2EREAL\_SUM\_IMAGE\ A.27a)\ \\ V0f)\ (ap\ (ap\ (c.2Epred\_set\_2EINSERT\ A.27a)\ V1e)\ V2s)) = (ap\ (ap \\ c.2Erealax\_2Ereal\_add\ (ap\ V0f\ V1e))\ (ap\ (ap\ (c.2Ereal\_sigma\_2EREAL\_SUM\_IMAGE \\ A.27a)\ V0f)\ (ap\ (ap\ (c.2Epred\_set\_2EDELETE\ A.27a)\ V2s)\ V1e)))))))))) \end{aligned} \quad (67)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (68)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (69)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (70)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (71)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (72)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (73)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (74)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (75)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (76)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (77)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (78)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (79)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (80)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (81)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (82)$$

**Theorem 1**

$$\begin{aligned} & \forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod \\ & (2^{A_{.27a}}) (ty\_2Epair\_2Eprod (2^{(2^{A_{.27a}})}) (ty\_2Erealx\_2Ereal^{(2^{A_{.27a}})}))) \\ & (\forall V1s \in (2^{A_{.27a}}).(((p (ap (c\_2Emeasure\_2Emeasure\_space \\ A_{.27a}) V0m)) \wedge ((p (ap (ap (c\_2Ebool\_2EIN (2^{A_{.27a}})) V1s) (ap (c\_2Emeasure\_2Emeasurable\_sets \\ A_{.27a}) V0m))) \wedge ((\forall V2x \in A_{.27a}.((p (ap (ap (c\_2Ebool\_2EIN \\ A_{.27a}) V2x) V1s)) \Rightarrow (p (ap (ap (c\_2Ebool\_2EIN (2^{A_{.27a}})) (ap (ap ( \\ c\_2Epred\_set\_2EINSERT A_{.27a}) V2x) (c\_2Epred\_set\_2EEMPTY A_{.27a}))) \\ (ap (c\_2Emeasure\_2Emeasurable\_sets A_{.27a}) V0m)))))) \wedge (p (ap ( \\ c\_2Epred\_set\_2EFINITE A_{.27a}) V1s)))))) \Rightarrow ((ap (ap (c\_2Emeasure\_2Emeasure \\ A_{.27a}) V0m) V1s) = (ap (ap (c\_2Ereal\_sigma\_2EREAL\_SUM\_IMAGE \\ A_{.27a}) (\lambda V3x \in A_{.27a}.(ap (ap (c\_2Emeasure\_2Emeasure A_{.27a}) \\ V0m) (ap (ap (c\_2Epred\_set\_2EINSERT A_{.27a}) V3x) (c\_2Epred\_set\_2EEMPTY \\ A_{.27a})))))) V1s)))))) \end{aligned}$$