

thm_2Emeasure_2EMEASURE_SPACE_REDUCE (TMQLQ6ma9PahryTR836955xy63nZgBKSGBt)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Emeasure_2Em_space : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Em_space\ A_27a \in ((2^{A-27a})(ty_2Epair_2Eprod\ (2^{A-27a})\ (ty_2Epair_2Eprod\ (2^{2^{A-27a}})\ (ty_2Erealax_2Ereal^{(2^{A-27a})})))) \tag{3}$$

Let $c_2Emeasure_2Emeasurable_sets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Emeasurable_sets\ A_27a \in ((2^{(2^{A-27a})})(ty_2Epair_2Eprod\ (2^{A-27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A-27a})})\ (ty_2Erealax_2Ereal^{(2^{A-27a})})))) \tag{4}$$

Let $c_2Emeasure_2Emeasure : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Emeasure\ A_27a \in (ty_2Erealax_2Ereal^{(2^{A-27a})})(ty_2Epair_2Eprod\ (2^{A-27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A-27a})})\ (ty_2Erealax_2Ereal^{(2^{A-27a})}))) \tag{5}$$

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A-27b})^{A-27a}}) \end{aligned} \quad (6)$$

Definition 7 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ **then** (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 9 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (9)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow (\forall V0sp \in (2^{A-27a}).(\forall V1sts \in \\ (2^{(2^{A-27a})}).(\forall V2mu \in (ty_2Erealax_2Ereal^{(2^{A-27a})}). \\ ((ap (c_2Emeasure_2Em_space A_27a) (ap (ap (c_2Epair_2E_2C (\\ 2^{A-27a}) (ty_2Epair_2Eprod (2^{(2^{A-27a})}) (ty_2Erealax_2Ereal^{(2^{A-27a})}))) \\ V0sp) (ap (ap (c_2Epair_2E_2C (2^{(2^{A-27a})}) (ty_2Erealax_2Ereal^{(2^{A-27a})}))) \\ V1sts) V2mu))) = V0sp))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow (\forall V0sp \in (2^{A-27a}).(\forall V1sts \in \\ (2^{(2^{A-27a})}).(\forall V2mu \in (ty_2Erealax_2Ereal^{(2^{A-27a})}). \\ ((ap (c_2Emeasure_2Emeasurable_sets A_27a) (ap (ap (c_2Epair_2E_2C \\ (2^{A-27a}) (ty_2Epair_2Eprod (2^{(2^{A-27a})}) (ty_2Erealax_2Ereal^{(2^{A-27a})}))) \\ V0sp) (ap (ap (c_2Epair_2E_2C (2^{(2^{A-27a})}) (ty_2Erealax_2Ereal^{(2^{A-27a})}))) \\ V1sts) V2mu))) = V1sts))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0sp \in (2^{A-27a}). (\forall V1sts \in \\
& \quad (2^{(2^{A-27a})}). (\forall V2mu \in (ty_2Erealax_2Ereal^{(2^{A-27a})}). \\
& ((ap\ (c_2Emeasure_2Emeasure\ A.27a)\ (ap\ (ap\ (c_2Epair_2E_2C\ (2^{A-27a}) \\
& \quad (ty_2Epair_2Eprod\ (2^{(2^{A-27a})})\ (ty_2Erealax_2Ereal^{(2^{A-27a})}))) \\
& V0sp)\ (ap\ (ap\ (c_2Epair_2E_2C\ (2^{(2^{A-27a})})\ (ty_2Erealax_2Ereal^{(2^{A-27a})}) \\
& \quad V1sts)\ V2mu))) = V2mu)))
\end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0x \in A.27a. (\forall V1y \in A.27b. (\forall V2a \in A.27a. (\forall V3b \in \\
& \quad A.27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27b)\ V0x)\ V1y) = (ap\ (ap \\
& (c_2Epair_2E_2C\ A.27a\ A.27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))
\end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0x \in (ty_2Epair_2Eprod\ A.27a\ A.27b). (\exists V1q \in A.27a. \\
& (\exists V2r \in A.27b. (V0x = (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27b) \\
& \quad V1q)\ V2r))))))
\end{aligned} \tag{14}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& (2^{A-27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A-27a})})\ (ty_2Erealax_2Ereal^{(2^{A-27a})}))). \\
& ((ap\ (ap\ (c_2Epair_2E_2C\ (2^{A-27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A-27a})}) \\
& \quad (ty_2Erealax_2Ereal^{(2^{A-27a})})))\ (ap\ (c_2Emeasure_2Em_space \\
& \quad A.27a)\ V0m))\ (ap\ (ap\ (c_2Epair_2E_2C\ (2^{(2^{A-27a})})\ (ty_2Erealax_2Ereal^{(2^{A-27a})}) \\
& (ap\ (c_2Emeasure_2Emeasurable_sets\ A.27a)\ V0m))\ (ap\ (c_2Emeasure_2Emeasure \\
& \quad A.27a)\ V0m))) = V0m))
\end{aligned}$$