

# thm\_2Emeasure\_2EMONOTONE\_CONVERGENCE2 (TMZ6hBSokHpc5Rbre1ieKGM YFGMwsXYvNDM)

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**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A$ . **if**  $(\exists x \in A.p (ap P x))$  **then** (the  $(\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ ).

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A$

**Definition 4** We define  $c\_2Ebool\_2E\_T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V$

Let  $ty\_2Erealx\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealx\_2Ereal \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $c\_2Emeasure\_2Emeasure : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Emeasure\ A\_27a \in ( (ty\_2Erealx\_2Ereal^{(2^{A\_27a})})(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})}))\ (ty\_2Erealx\_2Ereal^{(2^{A\_27a})})) \tag{3}$$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}$

**Definition 6** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{4}$$

**Definition 7** We define  $c\_2Ebool\_2E\_F$  to be  $(ap (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 8** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 9** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_7E$

**Definition 10** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21) 2) (\lambda V2t \in$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (5)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (6)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (7)$$

**Definition 11** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num$

**Definition 12** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 13** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 14** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21) 2) (\lambda V2t \in$

**Definition 15** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (8)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax}) \quad (9)$$

**Definition 16** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E\_40) (t$

Let  $c\_2Erealax\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{ty\_2Erealax}) \quad (10)$$

Let  $c\_2Erealax\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal}) \quad (11)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (12)$$

**Definition 17** We define  $c\_Erealax\_Ereal\_ABS$  to be  $\lambda V0r \in (ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)$

**Definition 18** We define  $c\_Erealax\_Ereal\_neg$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal.(ap\ c\_Erealax\_Ereal)$

Let  $c\_Erealax\_Etrealm\_add : \iota$  be given. Assume the following.

$$c\_Erealax\_Etrealm\_add \in (((ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)\ (ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal))\ (ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal))\ (ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal) \quad (13)$$

**Definition 19** We define  $c\_Erealax\_Ereal\_add$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal.\lambda V1T2 \in ty\_Erealax\_Ereal$

**Definition 20** We define  $c\_Ereal\_Ereal\_sub$  to be  $\lambda V0x \in ty\_Erealax\_Ereal.\lambda V1y \in ty\_Erealax\_Ereal$

Let  $c\_Eenum\_EZERO\_REP : \iota$  be given. Assume the following.

$$c\_Eenum\_EZERO\_REP \in \omega \quad (14)$$

**Definition 21** We define  $c\_Eenum\_E0$  to be  $(ap\ c\_Eenum\_EABS\_num\ c\_Eenum\_EZERO\_REP)$ .

Let  $c\_Ereal\_Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_Ereal\_Ereal\_of\_num \in (ty\_Erealax\_Ereal^{ty\_Eenum\_Eenum}) \quad (15)$$

Let  $c\_Erealax\_Etrealm\_lt : \iota$  be given. Assume the following.

$$c\_Erealax\_Etrealm\_lt \in ((2^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)}\ (ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal))\ (ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)) \quad (16)$$

**Definition 22** We define  $c\_Erealax\_Ereal\_lt$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal.\lambda V1T2 \in ty\_Erealax\_Ereal$

**Definition 23** We define  $c\_Ereal\_Ereal\_lte$  to be  $\lambda V0x \in ty\_Erealax\_Ereal.\lambda V1y \in ty\_Erealax\_Ereal$

**Definition 24** We define  $c\_Ebool\_ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

**Definition 25** We define  $c\_Ereal\_Eabs$  to be  $\lambda V0x \in ty\_Erealax\_Ereal.(ap\ (ap\ (ap\ (c\_Ebool\_ECOND$

Let  $c\_Epair\_EESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_Epair\_EESND\ A\_27a\ A\_27b \in (A\_27b)^{(ty\_Epair\_Eprod\ A\_27a\ A\_27b)} \quad (17)$$

Let  $c\_Epair\_EFAST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_Epair\_EFAST\ A\_27a\ A\_27b \in (A\_27a)^{(ty\_Epair\_Eprod\ A\_27a\ A\_27b)} \quad (18)$$

**Definition 26** We define  $c\_Epair\_EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c)^{A\_27a \times A\_27b})$



**Definition 32** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in$

**Definition 33** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap (c\_2Epred\_set\_2EIMAGE$

**Definition 34** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Epred\_set\_2EIMAGE$

Let  $c\_2Emeasure\_2Emeasurable\_sets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emeasure\_2Emeasurable\_sets A\_27a \in ((2^{(2^{A\_27a})})(ty\_2Epair\_2Eprod (2^{A\_27a}) (ty\_2Epair\_2Eprod (2^{(2^{A\_27a})}) (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))) \quad (27)$$

**Definition 35** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2ET)$ .

**Definition 36** We define  $c\_2Epred\_set\_2EFUNSET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0P \in (2^{A\_27a}).\lambda V1Q \in (2^{A\_27b}).(ap (c\_2Epred\_set\_2EIMAGE$

Let  $c\_2Ereal\_2Esum : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Esum \in ((ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})})(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum)) \quad (28)$$

**Definition 37** We define  $c\_2Eseq\_2Esums$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1s \in ty\_2Enum\_2Enum$

**Definition 38** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

**Definition 39** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Epred\_set\_2EIMAGE$

**Definition 40** We define  $c\_2Epred\_set\_2EDISJOINT$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Epred\_set\_2EIMAGE$

**Definition 41** We define  $c\_2Emeasure\_2Ecountably\_additive$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))$

**Definition 42** We define  $c\_2Emeasure\_2Epositive$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))$

Let  $c\_2Emeasure\_2Em\_space : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emeasure\_2Em\_space A\_27a \in ((2^{A\_27a})(ty\_2Epair\_2Eprod (2^{A\_27a}) (ty\_2Epair\_2Eprod (2^{(2^{A\_27a})}) (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))) \quad (29)$$

Let  $c\_2Emeasure\_2Esubsets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emeasure\_2Esubsets A\_27a \in (2^{(2^{A\_27a})})(ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})})) \quad (30)$$

**Definition 43** We define  $c\_2Epred\_set\_2EINJ$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^{A\_27a}).(ap (c\_2Epred\_set\_2EIMAGE$

**Definition 44** We define  $c\_2Epred\_set\_2Ecountable$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap (c\_2Ebool\_2E3F (c\_2Epred\_set\_2EIMAGE$

**Definition 45** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Epred\_set\_2EIMAGE$

Let  $c\_2Emeasure\_2Espace : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Espace\ A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))}) \quad (31)$$

**Definition 46** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2E$

**Definition 47** We define  $c\_2Emeasure\_2Esubset\_class$  to be  $\lambda A\_27a : \iota.\lambda V0sp \in (2^{A\_27a}).\lambda V1sts \in (2^{(2^{A\_27a})})$

**Definition 48** We define  $c\_2Emeasure\_2Ealgebra$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))$

**Definition 49** We define  $c\_2Emeasure\_2Esigma\_algebra$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))$

**Definition 50** We define  $c\_2Emeasure\_2Emeasure\_space$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))$

Assume the following.

$$True \quad (32)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (33)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (34)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (35)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \quad (36)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (37)$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod \\
& (2^{A_{.27a}}) (ty\_2Epair\_2Eprod (2^{(2^{A_{.27a}})}) (ty\_2Erealax\_2Ereal^{(2^{A_{.27a}})}))) \\
& (\forall V1s \in (2^{A_{.27a}}). (\forall V2f \in ((2^{A_{.27a}})^{ty\_2Enum\_2Enum}). \\
& (((p (ap (c\_2Emeasure\_2Emeasure\_space A_{.27a}) V0m)) \wedge ((p (ap ( \\
& ap (c\_2Ebool\_2EIN ((2^{A_{.27a}})^{ty\_2Enum\_2Enum})) V2f) (ap (ap (c\_2Epred\_set\_2EFUNSET \\
& ty\_2Enum\_2Enum (2^{A_{.27a}}) (c\_2Epred\_set\_2EUNIV ty\_2Enum\_2Enum)) \\
& (ap (c\_2Emeasure\_2Emeasurable\_sets A_{.27a}) V0m)))) \wedge ((\forall V3n \in \\
& ty\_2Enum\_2Enum. (p (ap (ap (c\_2Epred\_set\_2ESUBSET A_{.27a}) (ap \\
& V2f V3n)) (ap V2f (ap c\_2Enum\_2ESUC V3n)))))) \wedge (V1s = (ap (c\_2Epred\_set\_2EBIGUNION \\
& A_{.27a}) (ap (ap (c\_2Epred\_set\_2EIMAGE ty\_2Enum\_2Enum (2^{A_{.27a}}) \\
& V2f) (c\_2Epred\_set\_2EUNIV ty\_2Enum\_2Enum)))))) \Rightarrow (p (ap (ap \\
& c\_2Eseq\_2E\_2D\_2D\_3E (ap (ap (c\_2Ecombin\_2Eo ty\_2Enum\_2Enum ty\_2Erealax\_2Ereal \\
& (2^{A_{.27a}}) (ap (c\_2Emeasure\_2Emeasure A_{.27a}) V0m)) V2f)) (ap ( \\
& ap (c\_2Emeasure\_2Emeasure A_{.27a}) V0m) V1s))))))
\end{aligned} \tag{38}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{39}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow \\
& (((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False}))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False}))))))
\end{aligned} \tag{42}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee \neg(p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee \neg(p V0p))))))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge ( \\
& \neg(p V1q) \vee ((p V2r) \vee \neg(p V0p))))))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q)) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge (\neg(p V1q) \vee \neg(p V0p))))))
\end{aligned} \tag{48}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q)) \Rightarrow (p V0p))) \tag{49}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q)) \Rightarrow \neg(p V1q))) \tag{50}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a. \text{nonempty } A\_27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod \\
& (2^{A\_27a}) (ty\_2Epair\_2Eprod (2^{(2^{A\_27a})}) (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))). \\
& (\forall V1f \in ((2^{A\_27a})^{ty\_2Enum\_2Enum}). (((p (ap (c\_2Emeasure\_2Emeasure\_space \\
& A\_27a) V0m)) \wedge ((p (ap (ap (c\_2Ebool\_2EIN ((2^{A\_27a})^{ty\_2Enum\_2Enum})) \\
& V1f) (ap (ap (c\_2Epred\_set\_2EFUNSET ty\_2Enum\_2Enum (2^{A\_27a})) \\
& (c\_2Epred\_set\_2EUNIV ty\_2Enum\_2Enum)) (ap (c\_2Emeasure\_2Emeasurable\_sets \\
& A\_27a) V0m)))))) \wedge (\forall V2n \in ty\_2Enum\_2Enum. (p (ap (ap (c\_2Epred\_set\_2ESUBSET \\
& A\_27a) (ap V1f V2n)) (ap V1f (ap c\_2Enum\_2ESUC V2n)))))) \Rightarrow (p (ap \\
& (ap c\_2Eseq\_2E\_2D\_2D\_3E (ap (ap (c\_2Ecombin\_2Eo ty\_2Enum\_2Enum \\
& ty\_2Erealax\_2Ereal (2^{A\_27a})) (ap (c\_2Emeasure\_2Emeasure A\_27a) \\
& V0m)) V1f)) (ap (ap (c\_2Emeasure\_2Emeasure A\_27a) V0m) (ap (c\_2Epred\_set\_2EBIGUNION \\
& A\_27a) (ap (ap (c\_2Epred\_set\_2EIMAGE ty\_2Enum\_2Enum (2^{A\_27a})) \\
& V1f) (c\_2Epred\_set\_2EUNIV ty\_2Enum\_2Enum)))))))))
\end{aligned}$$