

thm\_2Emeasure\_2ESIGMA\_ALGEBRA\_ALT\_DISJOINT  
 (TMH5Nbj8UmnES1HN9FYYSPtXcFZiVEMUwxa)

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Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A_0.\text{nonempty } A_0 \Rightarrow \forall A_1.\text{nonempty } A_1 \Rightarrow \text{nonempty } (ty\_2Epair\_2Eprod \\ A_0 A_1) \end{aligned} \tag{1}$$

Let  $c\_2Emeasure\_2Esubsets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow c\_2Emeasure\_2Esubsets A\_27a \in (2^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))} \tag{2}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. (\lambda V1f \in (2^{A\_27a}). (ap V1f V0x)))$

**Definition 3** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 6** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2. inj\_o (t1 = t2))))$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2. inj\_o (t1 = t2))))$

Let  $c\_2Epair\_2EAbs\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow c\_2Epair\_2EAbs\_prod \\ A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \tag{3}$$

**Definition 8** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b. (ap (c\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}})$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A_27a \ A_27b \in ((2^{A_27a})^{(ty\_2Epair\_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (4)$$

**Definition 9** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c\_2$

Let  $c\_2Emeasure\_2Espace : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c\_2Emeasure\_2Espace A_27a \in ((2^{A_27a})^{(ty\_2Epair\_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))}) \quad (5)$$

**Definition 10** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E_21\ 2)) (\lambda V0t \in 2. V0t)$ .

**Definition 11** We define  $c\_2Ebool\_2E_7E$  to be  $(\lambda V0t \in 2. (ap (ap c\_2Emin\_2E_3D\_3D\_3E\ V0t)) c\_2Ebool\_2E_7E)$

**Definition 12** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c\_2$

**Definition 13** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c\_2Ebool\_2EF)$ .

**Definition 14** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c\_2$

**Definition 15** We define  $c\_2Emeasure\_2Esubset\_class$  to be  $\lambda A_27a : \iota. \lambda V0sp \in (2^{A_27a}). \lambda V1sts \in (2^{(2^{A_27a})})$

**Definition 16** We define  $c\_2Emeasure\_2Ealgebra$  to be  $\lambda A_27a : \iota. \lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))$

**Definition 17** We define  $c\_2Emin\_2E_40$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p$

**Definition 18** We define  $c\_2Ebool\_2E_3F$  to be  $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c\_2Emin\_2E_40$

**Definition 19** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^{A_27a})}). (ap (c\_2Epred\_set\_2$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$\text{nonempty } ty\_2Enum\_2Enum \quad (6)$$

**Definition 20** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c\_2Ebool\_2ET)$ .

**Definition 21** We define  $c\_2Epred\_set\_2EINJ$  to be  $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (2^{A_27b})$

**Definition 22** We define  $c\_2Epred\_set\_2Ecountable$  to be  $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (ap (c\_2Ebool\_2E_3F$

**Definition 23** We define  $c\_2Emeasure\_2Esigma\_algebra$  to be  $\lambda A_27a : \iota. \lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))$

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $c\_2Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (8)$$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (9)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (10)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (11)$$

**Definition 24** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num$

**Definition 25** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 26** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 27** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (12)$$

**Definition 28** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

**Definition 29** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.$

**Definition 30** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap (ap (ap (ap (c\_2Ebool\_2E$

Let  $c\_2Earithmetic\_2EEEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (13)$$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (14)$$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (15)$$

**Definition 31** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

**Definition 32** We define  $c\_2Enumeral\_2EiiSUC$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap c\_2Enum\_2ESUC (ap$

**Definition 33** We define  $c\_2Enumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (16)$$

**Definition 34** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap (ap c\_2Earithmetic$

**Definition 35** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap (ap c\_2Earithmetic$

**Definition 36** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

**Definition 37** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. (ap$

**Definition 38** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap (c\_2Epred\_set$

**Definition 39** We define  $c\_2Epred\_set\_2EDISJOINT$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap (c\_2Epred\_set$

**Definition 40** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (A\_27b^{A\_27a}). \lambda V1s \in (A\_27b^{A\_27a}). (ap (c\_2Epred\_set$

**Definition 41** We define  $c\_2Epred\_set\_2EFUNSET$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0P \in (2^{A\_27a}). \lambda V1Q \in (2^{A\_27a}). (ap (c\_2Epred\_set$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2B V0m) c\_2Enum\_2E0) = V0m)) \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & ((ap (ap c\_2Earithmetic\_2E\_2B c\_2Enum\_2E0) V0m) = V0m) \wedge (((ap ( \\ & ap c\_2Earithmetic\_2E\_2B V0m) c\_2Enum\_2E0) = V0m) \wedge (((ap (ap c\_2Earithmetic\_2E\_2B \\ & (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B \\ & V0m) V1n))) \wedge ((ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap c\_2Enum\_2ESUC \\ & V1n)) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n))))))) \\ & \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n) = (ap (ap c\_2Earithmetic\_2E\_2B \\ & V1n) V0m)))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
 & \forall V2p \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2B V0m) \\
 & (ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p)) = (ap (ap c\_2Earithmetic\_2E\_2B \\
 & (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) V2p)))) \\
 \end{aligned} \tag{20}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. ((V0m = c\_2Enum\_2E0) \vee (\exists V1n \in ty\_2Enum\_2Enum. (V0m = (ap c\_2Enum\_2ESUC V1n))))) \tag{21}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
 & ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge \\
 & (((ap (ap c\_2Earithmetic\_2E\_2A V0m) c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge \\
 & (((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL \\
 & (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0m) = V0m) \wedge \\
 & (((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Earithmetic\_2ENUMERAL \\
 & (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = V0m) \wedge \\
 & ((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap \\
 & (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)) \\
 & V1n)) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Enum\_2ESUC V1n)) = \\
 & (ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap (ap c\_2Earithmetic\_2E\_2A \\
 & V0m) V1n))))))) \\
 \end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
 & \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 & (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) (ap (ap c\_2Earithmetic\_2E\_2B \\
 & V0m) V2p)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))))) \\
 \end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
 & (\neg(V0m = V1n)) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC \\
 & V0m)) V1n)) \vee (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC \\
 & V1n)) V0m))))) \\
 \end{aligned} \tag{24}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. ((ap c\_2Enum\_2ESUC V0n) = (ap (ap \\
 c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
 c\_2Earithmetic\_2EZERO))) V0n))) \tag{25}$$

Assume the following.

$$True \quad (26)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (29)$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a. (p V0t) \Leftrightarrow (p V0t))) \quad (30)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge (p V1t2) \wedge (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (31)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (32)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (33)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (34)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (35)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow (\neg(p V0t)))))) \quad (36)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (37)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (38)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (39)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (40)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\forall V1x \in A\_27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A\_27a.(\neg(p (ap V0P V2x))))))) \quad (41)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\exists V1x \in A\_27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A\_27a.(\neg(p (ap V0P V2x))))))) \quad (42)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((\forall V3x \in A\_27a.(p (ap V0P V3x))) \wedge (\forall V4x \in A\_27a.(p (ap V1Q V4x))))))) \quad (43)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p V0P) \vee (\exists V2x \in A\_27a.(p (ap V1Q V2x)))) \Leftrightarrow (\exists V3x \in A\_27a.((p V0P) \vee (p (ap V1Q V3x))))))) \quad (44)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).((\exists V2x \in A\_27a.((p V0P) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \wedge (\exists V3x \in A\_27a.(p (ap V1Q V3x))))))) \quad (45)$$

Assume the following.

$$\forall A. \text{A\_27a\_nonempty } A \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_{-27a}}). ((\forall V2x \in A. \text{A\_27a}. ((p \text{ V0P}) \vee (p \text{ (ap V1Q V2x))))))) \Leftrightarrow ((p \text{ V0P}) \vee (\forall V3x \in A. \text{A\_27a}. (p \text{ (ap V1Q V3x))))))) \quad (46)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p \text{ V0A}) \vee (p \text{ V1B})) \vee (p \text{ V2C}))) \Leftrightarrow (((p \text{ V0A}) \vee (p \text{ V1B})) \vee (p \text{ V2C})))) \quad (47)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p \text{ V0A}) \vee (p \text{ V1B})) \Leftrightarrow ((p \text{ V1B}) \vee (p \text{ V0A})))) \quad (48)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(p \text{ V0A}) \wedge (p \text{ V1B}))) \Leftrightarrow ((\neg(p \text{ V0A}) \vee (\neg(p \text{ V1B})))) \wedge ((\neg(p \text{ V0A}) \vee (p \text{ V1B}))) \Leftrightarrow ((\neg(p \text{ V0A}) \wedge (\neg(p \text{ V1B}))))))) \quad (49)$$

Assume the following.

$$(\forall V0t \in 2. (((p \text{ V0t}) \Rightarrow \text{False}) \Leftrightarrow ((p \text{ V0t}) \Leftrightarrow \text{False}))) \quad (50)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p \text{ V0t1}) \Rightarrow ((p \text{ V1t2}) \Rightarrow (p \text{ V2t3}))) \Leftrightarrow (((p \text{ V0t1}) \wedge (p \text{ V1t2})) \Rightarrow (p \text{ V2t3})))))) \quad (51)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_{-27} \in 2. (\forall V2y \in 2. (\forall V3y_{-27} \in 2. (((p \text{ V0x}) \Leftrightarrow (p \text{ V1x}_{-27})) \wedge ((p \text{ V1x}_{-27}) \Rightarrow ((p \text{ V2y}) \Leftrightarrow (p \text{ V3y}_{-27}))))))) \Rightarrow (((p \text{ V0x}) \Rightarrow (p \text{ V2y})) \Leftrightarrow ((p \text{ V1x}_{-27}) \Rightarrow (p \text{ V3y}_{-27})))))) \quad (52)$$

Assume the following.

$$\begin{aligned} \forall A. \text{A\_27a\_nonempty } A \Rightarrow & \forall A. \text{A\_27b\_nonempty } A \Rightarrow ( \\ & \forall V0P \in ((2^{A_{-27b}})^{A_{-27a}}). ((\forall V1x \in A. \text{A\_27a}. (\exists V2y \in (2^{A_{-27a}}). ((\forall V3f \in (A. \text{A\_27b}^{A_{-27a}}). ((\forall V4x \in A. \text{A\_27a}. (p \text{ (ap (ap V0P V1x) V2y)))) \Leftrightarrow (\exists V3f \in (A. \text{A\_27b}^{A_{-27a}}). ((\forall V4x \in A. \text{A\_27a}. (p \text{ (ap (ap V0P V4x) (ap V3f V4x))))))))))) \quad (53) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A. \text{A\_27a\_nonempty } A \Rightarrow & (\forall V0a \in (ty. \text{2Epair\_2Eprod} \\ & (2^{A_{-27a}}) (2^{(2^{A_{-27a}})})). ((\forall V1s \in (2^{A_{-27a}}). (\forall V2t \in (2^{A_{-27a}}). (((p \text{ (ap (c. \text{2Emeasure\_2Ealgebra} A. \text{27a}) V0a)}) \wedge ((p \text{ (ap (c. \text{2Ebool\_2EIN} (2^{A_{-27a}})) V1s}) \text{ (ap (c. \text{2Emeasure\_2Esubsets} \\ & A. \text{27a}) V0a))) \wedge ((p \text{ (ap (ap (c. \text{2Ebool\_2EIN} (2^{A_{-27a}})) V2t) \text{ (ap (c. \text{2Emeasure\_2Esubsets} \\ & A. \text{27a}) V0a)))) \Rightarrow ((p \text{ (ap (ap (c. \text{2Ebool\_2EIN} (2^{A_{-27a}})) (ap (ap (c. \text{2Epred\_set\_2EDIFF} \\ & A. \text{27a}) V1s) V2t)) \text{ (ap (c. \text{2Emeasure\_2Esubsets} A. \text{27a}) V0a))))))))))) \quad (54) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0a \in (ty\_2Epair\_2Eprod \\
& (2^{A_{27a}}) (2^{(2^{A_{27a}})})).((p (ap (c\_2Emeasure\_2Esigma\_algebra \\
& A_{27a}) V0a)) \Leftrightarrow ((p (ap (c\_2Emeasure\_2Ealgebra A_{27a}) V0a)) \wedge (\forall V1f \in \\
& ((2^{A_{27a}})^{ty\_2Enum\_2Enum}).((p (ap (ap (c\_2Ebool\_2EIN ((2^{A_{27a}})^{ty\_2Enum\_2Enum}) \\
& V1f) (ap (ap (c\_2Epred\_set\_2EFUNSET ty\_2Enum\_2Enum (2^{A_{27a}}) \\
& (c\_2Epred\_set\_2EUNIV ty\_2Enum\_2Enum)) (ap (c\_2Emeasure\_2Esubsets \\
& A_{27a}) V0a)))) \Rightarrow (p (ap (ap (c\_2Ebool\_2EIN (2^{A_{27a}})) (ap (c\_2Epred\_set\_2EBIGUNION \\
& A_{27a}) (ap (ap (c\_2Epred\_set\_2EIMAGE ty\_2Enum\_2Enum (2^{A_{27a}}) \\
& V1f) (c\_2Epred\_set\_2EUNIV ty\_2Enum\_2Enum)))) (ap (c\_2Emeasure\_2Esubsets \\
& A_{27a}) V0a))))))) \\
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0a \in (ty\_2Epair\_2Eprod \\
& (2^{A_{27a}}) (2^{(2^{A_{27a}})})).((p (ap (c\_2Emeasure\_2Esigma\_algebra \\
& A_{27a}) V0a)) \Leftrightarrow ((p (ap (c\_2Emeasure\_2Ealgebra A_{27a}) V0a)) \wedge (\forall V1f \in \\
& ((2^{A_{27a}})^{ty\_2Enum\_2Enum}).((p (ap (ap (c\_2Ebool\_2EIN ((2^{A_{27a}})^{ty\_2Enum\_2Enum}) \\
& V1f) (ap (ap (c\_2Epred\_set\_2EFUNSET ty\_2Enum\_2Enum (2^{A_{27a}}) \\
& (c\_2Epred\_set\_2EUNIV ty\_2Enum\_2Enum)) (ap (c\_2Emeasure\_2Esubsets \\
& A_{27a}) V0a)))) \wedge (((ap V1f c\_2Enum\_2E0) = (c\_2Epred\_set\_2EEMPTY \\
& A_{27a})) \wedge (\forall V2n \in ty\_2Enum\_2Enum.(p (ap (ap (c\_2Epred\_set\_2ESUBSET \\
& A_{27a}) (ap V1f V2n)) (ap V1f (ap c\_2Enum\_2ESUC V2n))))))) \Rightarrow (p (ap \\
& (ap (c\_2Ebool\_2EIN (2^{A_{27a}})) (ap (c\_2Epred\_set\_2EBIGUNION \\
& A_{27a}) (ap (ap (c\_2Epred\_set\_2EIMAGE ty\_2Enum\_2Enum (2^{A_{27a}}) \\
& V1f) (c\_2Epred\_set\_2EUNIV ty\_2Enum\_2Enum)))) (ap (c\_2Emeasure\_2Esubsets \\
& A_{27a}) V0a))))))) \\
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p (ap V0P c\_2Enum\_2E0)) \wedge \\
& (\forall V1n \in ty\_2Enum\_2Enum.((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c\_2Enum\_2ESUC \\
& V1n))))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p (ap V0P V2n)))) \\
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
& (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\
ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& (ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL \\
V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enumeral\_2EiZ (ap \\
& (ap c\_2Earithmetic\_2E\_2B V2n) V3m))))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\
& ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\
& ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\
& ((\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A ( \\
& ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL \\
V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A \\
V6n) V7m))))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\
& ((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\
ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL \\
V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D \\
V10n) V11m))))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\
c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
& (ap c\_2Earithmetic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
(ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\
ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V14n) c\_2Enum\_2E0) = \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\
& ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
& ((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) \\
& (ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap (ap c\_2Earithmetic\_2EEXP V15n) V16m))))))) \wedge (((ap c\_2Enum\_2ESUC \\
c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
c\_2Earithmetic\_2EZERO)))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum. \\
& (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Enum\_2ESUC V17n)))) \wedge (((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\
c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE \\
& (ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Eprim\_rec\_2EPRE V18n))))))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\
& (((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO)))) \wedge \\
& ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL \\
V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\
& (\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL \\
V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
V24n)))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL \\
V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
V25n) V26m))))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E \\
c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
V30m) V29n)))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL \\
V32n)))) \Leftrightarrow False)))) \wedge ((\forall V33n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL \\
V33n)) (ap c\_2Earithmetic\_2ENUMERAL V33n))) \Leftrightarrow True)))
\end{aligned}$$

Assume the following.

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D c_2Earithmetic_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Earithmetic_2EBIT2 V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (\neg(p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V1m) V0n))) \wedge ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))))))))))) \\
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in \\
& (2^{A\_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A\_27a. ((p (ap (ap (c_2Ebool\_2EIN \\
& A\_27a) V2x) V0s)) \Leftrightarrow (p (ap (ap (c_2Ebool\_2EIN A\_27a) V2x) V1t))))))) \\
\end{aligned} \tag{61}$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a. (\neg(p (ap (ap \\
(c_2Ebool\_2EIN A\_27a) V0x) (c_2Epred\_set\_2EEMPTY A\_27a))))) \tag{62}$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a. (p (ap (ap (c_2Ebool\_2EIN \\
A\_27a) V0x) (c_2Epred\_set\_2EUNIV A\_27a)))) \tag{63}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in \\
& (2^{A\_27a}). (\forall V2u \in (2^{A\_27a}). (((p (ap (ap (c_2Epred\_set\_2ESUBSET \\
& A\_27a) V0s) V1t)) \wedge (p (ap (ap (c_2Epred\_set\_2ESUBSET A\_27a) V1t) \\
& V2u))) \Rightarrow (p (ap (ap (c_2Epred\_set\_2ESUBSET A\_27a) V0s) V2u))))))) \\
\end{aligned} \tag{64}$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (p (ap ( \\
ap (c_2Epred\_set\_2ESUBSET A\_27a) V0s) V0s))) \tag{65}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in \\
& (2^{A\_27a}). (\forall V2x \in A\_27a. ((p (ap (ap (c_2Ebool\_2EIN A\_27a) \\
& V2x) (ap (ap (c_2Epred\_set\_2EINTER A\_27a) V0s) V1t))) \Leftrightarrow ((p (ap ( \\
ap (c_2Ebool\_2EIN A\_27a) V2x) V0s)) \wedge (p (ap (ap (c_2Ebool\_2EIN \\
& A\_27a) V2x) V1t))))))) \\
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & (\forall V0s \in (2^{A\_27a}).(\forall V1t \in \\ (2^{A\_27a}).((p (ap (ap (c\_2Epred\_set\_2EDISJOINT A\_27a) V0s) V1t)) \Leftrightarrow & (67) \\ (p (ap (ap (c\_2Epred\_set\_2EDISJOINT A\_27a) V1t) V0s)))))) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & (\forall V0s \in (2^{A\_27a}).(\forall V1t \in \\ (2^{A\_27a}).(\forall V2x \in A\_27a.((p (ap (ap (c\_2Ebool\_2EIN A\_27a) \\ V2x) (ap (ap (c\_2Epred\_set\_2EDIFF A\_27a) V0s) V1t))) \Leftrightarrow ((p (ap ( \\ ap (c\_2Ebool\_2EIN A\_27a) V2x) V0s)) \wedge (\neg(p (ap (ap (c\_2Ebool\_2EIN \\ A\_27a) V2x) V1t)))))))))) \end{aligned} \quad (68)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & \forall A\_27b.\text{nonempty } A\_27b \Rightarrow ( \\ \forall V0y \in A\_27b.(\forall V1s \in (2^{A\_27a}).(\forall V2f \in (A\_27b^{A\_27a}). \\ ((p (ap (ap (c\_2Ebool\_2EIN A\_27b) V0y) (ap (ap (c\_2Epred\_set\_2EIMAGE \\ A\_27a A\_27b) V2f) V1s))) \Leftrightarrow (\exists V3x \in A\_27a.((V0y = (ap V2f V3x)) \wedge \\ (p (ap (ap (c\_2Ebool\_2EIN A\_27a) V3x) V1s)))))))) \end{aligned} \quad (69)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & \forall A\_27b.\text{nonempty } A\_27b \Rightarrow ( \\ \forall V0f \in (A\_27b^{A\_27a}).(\forall V1P \in (2^{A\_27a}).(\forall V2Q \in \\ (2^{A\_27b}).((p (ap (ap (c\_2Ebool\_2EIN (A\_27b^{A\_27a})) V0f) (ap (ap \\ (c\_2Epred\_set\_2EFUNSET A\_27a A\_27b) V1P) V2Q))) \Leftrightarrow (\forall V3x \in \\ A\_27a.((p (ap (ap (c\_2Ebool\_2EIN A\_27a) V3x) V1P)) \Rightarrow (p (ap (ap (c\_2Ebool\_2EIN \\ A\_27b) (ap V0f V3x)) V2Q)))))))) \end{aligned} \quad (70)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & (\forall V0x \in A\_27a.(\forall V1sos \in \\ (2^{(2^{A\_27a})}).((p (ap (ap (c\_2Ebool\_2EIN A\_27a) V0x) (ap (c\_2Epred\_set\_2EBIGUNION \\ A\_27a) V1sos))) \Leftrightarrow (\exists V2s \in (2^{A\_27a}).((p (ap (ap (c\_2Ebool\_2EIN \\ A\_27a) V0x) V2s)) \wedge (p (ap (ap (c\_2Ebool\_2EIN (2^{A\_27a})) V2s) V1sos)))))))) \end{aligned} \quad (71)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (72)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (73)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow \\ ((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))) \quad (74)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (75)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (76)$$

Assume the following.

$$\begin{aligned} &(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (77)$$

Assume the following.

$$\begin{aligned} &(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee ((\neg(p V0p)) \wedge ((p V2r) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (78)$$

Assume the following.

$$\begin{aligned} &(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \wedge ((p V0p) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee ((p V2r) \vee ((\neg(p V0p))))))))))) \end{aligned} \quad (79)$$

Assume the following.

$$\begin{aligned} &(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \wedge ((p V0p) \vee ((\neg(p V2r)) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee ((\neg(p V0p))))))))))) \end{aligned} \quad (80)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (81)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))))) \quad (82)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (83)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (84)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q))))) \quad (85)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (86)$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in ((2^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}).(((\forall V1m \in \\ & ty\_2Enum\_2Enum. (\forall V2n \in ty\_2Enum\_2Enum. ((p (ap (ap V0P V1m) \\ & V2n)) \Rightarrow (p (ap (ap V0P V2n) V1m)))))) \wedge (\forall V3m \in ty\_2Enum\_2Enum. \\ & (\forall V4n \in ty\_2Enum\_2Enum. (p (ap (ap V0P V3m) (ap (ap c\_2Earithmetic\_2E\_2B \\ & V3m) V4n)))))) \Rightarrow (\forall V5m \in ty\_2Enum\_2Enum. (\forall V6n \in ty\_2Enum\_2Enum. \\ & (p (ap (ap V0P V5m) V6n)))))) \end{aligned} \quad (87)$$

### Theorem 1

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0a \in (ty\_2Epair\_2Eprod \\ & (2^{A\_27a}) (2^{(2^{A\_27a})})).((p (ap (c\_2Emeasure\_2Esigma\_algebra \\ & A\_27a) V0a)) \Leftrightarrow ((p (ap (c\_2Emeasure\_2Ealgebra A\_27a) V0a)) \wedge (\forall V1f \in \\ & ((2^{A\_27a})^{ty\_2Enum\_2Enum}).(((p (ap (ap (c\_2Ebool\_2EIN ((2^{A\_27a})^{ty\_2Enum\_2Enum})) \\ & V1f) (ap (ap (c\_2Epred\_set\_2EFUNSET ty\_2Enum\_2Enum (2^{A\_27a})) \\ & (c\_2Epred\_set\_2EUNIV ty\_2Enum\_2Enum)) (ap (c\_2Emeasure\_2Esubsets \\ & A\_27a) V0a)))) \wedge (\forall V2m \in ty\_2Enum\_2Enum. (\forall V3n \in ty\_2Enum\_2Enum. \\ & ((\neg(V2m = V3n)) \Rightarrow (p (ap (ap (c\_2Epred\_set\_2EDISJOINT A\_27a) (ap \\ & V1f V2m)) (ap (V1f V3n)))))) \Rightarrow (p (ap (ap (c\_2Ebool\_2EIN (2^{A\_27a})) \\ & (ap (c\_2Epred\_set\_2EBIGUNION A\_27a) (ap (ap (c\_2Epred\_set\_2EIMAGE \\ & ty\_2Enum\_2Enum (2^{A\_27a})) V1f) (c\_2Epred\_set\_2EUNIV ty\_2Enum\_2Enum)))) \\ & (ap (c\_2Emeasure\_2Esubsets A\_27a) V0a)))))))))) \end{aligned}$$