

thm\_2Emeasure\_2ESIGMA\_\_PROPERTY\_\_ALT  
(TMGh-  
SWN3kSfiRzxDrTMFaCvfgnMCfQBhKjV)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2E\_2T)$ .

**Definition 4** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 5** We define  $c\_2Ebool\_2E\_2IN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A-27a}).(ap V1f V0x)))$

**Definition 6** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a})))$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

**Definition 8** We define  $c\_2Epred\_set\_2EINJ$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A-27a}).\lambda V1s \in (2^{A-27a}).$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p))$  of type  $\iota \Rightarrow \iota$ .

**Definition 10** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c\_2Emin\_2E\_40$

**Definition 11** We define  $c\_2Epred\_set\_2Ecountable$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).(ap (c\_2Ebool\_2E\_3F$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $c\_2Emeasure\_2Esubsets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Esubsets\ A\_27a \in ( (2^{(2^{A-27a})}) (ty\_2Epair\_2Eprod\ (2^{A-27a})\ (2^{(2^{A-27a})}))) \tag{3}$$

**Definition 12** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.(ty\_2Epair\_2EABS\_prod \iota \Rightarrow \iota \Rightarrow \iota)))))$ .  
Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (4)$$

**Definition 13** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epred\_set\_2EGSPEC \iota \Rightarrow \iota \Rightarrow \iota))$ .  
Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}}) \end{aligned} \quad (5)$$

**Definition 14** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Emeasure\_2Espace \iota \Rightarrow \iota))$ .  
Let  $c\_2Emeasure\_2Espace : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emeasure\_2Espace A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))}) \quad (6)$$

**Definition 15** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 16** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_21 2))$ .

**Definition 17** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Epred\_set\_2E\_21 2) s t))$ .

**Definition 18** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

**Definition 19** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Epred\_set\_2E\_21 2) s t))$ .

**Definition 20** We define  $c\_2Emeasure\_2Esubset\_class$  to be  $\lambda A\_27a : \iota.\lambda V0sp \in (2^{A\_27a}).\lambda V1sts \in (2^{(2^{A\_27a})})$ .

**Definition 21** We define  $c\_2Emeasure\_2Ealgebra$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))$ .

**Definition 22** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap (c\_2Epred\_set\_2E\_21 2) P))$ .

**Definition 23** We define  $c\_2Emeasure\_2Esigma\_algebra$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))$ .

**Definition 24** We define  $c\_2Epred\_set\_2EBIGINTER$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap (c\_2Epred\_set\_2E\_21 2) P))$ .

**Definition 25** We define  $c\_2Emeasure\_2Esigma$  to be  $\lambda A\_27a : \iota.\lambda V0sp \in (2^{A\_27a}).\lambda V1st \in (2^{(2^{A\_27a})}).(ap (c\_2Emeasure\_2Esubset\_class A\_27a sp) st))$ .

**Definition 26** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b)^{A\_27a}.\lambda V1s \in (A\_27b)$ .

**Definition 27** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Epred\_set\_2E\_21 2) s t))$ .

**Definition 28** We define  $c\_2Epred\_set\_2EFUNSET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0P \in (2^{A\_27a}).\lambda V1Q \in (2^{A\_27b})$ .

Assume the following.

$$True \quad (7)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (8)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (9)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (14)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (15)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (17)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\forall V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p (ap V0P V2x))))))) \quad (18)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).(((p V0P) \wedge (\forall V2x \in A.27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A.27a.((p V0P) \wedge (p (ap V1Q V3x))))))) \quad (19)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).(((p V0P) \vee (\exists V2x \in A.27a.(p (ap V1Q V2x)))) \Leftrightarrow (\exists V3x \in A.27a.((p V0P) \vee (p (ap V1Q V3x))))))) \quad (20)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in 2.((\exists V2x \in A.27a.((p (ap V0P V2x)) \wedge (p V1Q))) \Leftrightarrow ((\exists V3x \in A.27a.(p (ap V0P V3x))) \wedge (p V1Q)))))) \quad (21)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).(((\forall V2x \in A.27a.((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A.27a.(p (ap V1Q V3x))))))) \quad (22)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (23)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (24)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B))))))) \quad (25)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (26)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x.27 \in 2.(\forall V2y \in 2.(\forall V3y.27 \in 2.(((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \quad (27)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A-27a}). (\forall V1a \in A\_27a. ((\exists V2x \in A\_27a. ((V2x = V1a) \wedge (p (ap\ V0P\ V2x)))) \Leftrightarrow (p (ap\ V0P\ V1a)))))) \quad (28)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0P \in ((2^{A-27b})^{A-27a}). ((\forall V1x \in A\_27a. (\exists V2y \in A\_27b. (p (ap (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (\exists V3f \in (A\_27b^{A-27a}). (\forall V4x \in A\_27a. (p (ap (ap\ V0P\ V4x)\ (ap\ V3f\ V4x))))))) \quad (29)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (2^{A-27a}). (\forall V1y \in (2^{(2^{A-27a})}). ((ap (c\_2Emeasure\_2Espace\ A\_27a) (ap (ap (c\_2Epair\_2E\_2C (2^{A-27a}) (2^{(2^{A-27a})}))\ V0x)\ V1y)) = V0x))) \quad (30)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (2^{A-27a}). (\forall V1y \in (2^{(2^{A-27a})}). ((ap (c\_2Emeasure\_2Esubsets\ A\_27a) (ap (ap (c\_2Epair\_2E\_2C (2^{A-27a}) (2^{(2^{A-27a})}))\ V0x)\ V1y)) = V1y))) \quad (31)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in (ty\_2Epair\_2Eprod (2^{A-27a}) (2^{(2^{A-27a})})). ((p (ap (c\_2Emeasure\_2Ealgebra\ A\_27a) V0a)) \Rightarrow (p (ap (ap (c\_2Ebool\_2EIN (2^{A-27a}) (c\_2Epred\_set\_2EEMPTY A\_27a) (ap (c\_2Emeasure\_2Esubsets\ A\_27a) V0a)))))) \quad (32)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in (ty\_2Epair\_2Eprod (2^{A-27a}) (2^{(2^{A-27a})})). (\forall V1s \in (2^{A-27a}). (((p (ap (c\_2Emeasure\_2Ealgebra\ A\_27a) V0a)) \wedge (p (ap (ap (c\_2Ebool\_2EIN (2^{A-27a}) V1s) (ap (c\_2Emeasure\_2Esubsets\ A\_27a) V0a)))) \Rightarrow (p (ap (ap (c\_2Ebool\_2EIN (2^{A-27a}) (ap (ap (c\_2Epred\_set\_2EDIFF A\_27a) (ap (c\_2Emeasure\_2Espace\ A\_27a) V0a))\ V1s)) (ap (c\_2Emeasure\_2Esubsets\ A\_27a) V0a)))))) \quad (33)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in (ty\_2Epair\_2Eprod (2^{A-27a}) (2^{(2^{A-27a})})). ((p (ap (c\_2Emeasure\_2Esigma\_algebra\ A\_27a) V0a)) \Rightarrow (p (ap (c\_2Emeasure\_2Ealgebra\ A\_27a) V0a)))) \quad (34)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0sp \in (2^{A\_27a}). (\forall V1sts \in \\ (2^{(2^{A\_27a})}). ((p (ap (ap (c\_2Emeasure\_2Esubset\_class\ A\_27a) \\ V0sp)\ V1sts)) \Rightarrow (p (ap (c\_2Emeasure\_2Esigma\_algebra\ A\_27a) (ap \\ (ap (c\_2Emeasure\_2Esigma\ A\_27a)\ V0sp)\ V1sts)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0sp \in (2^{A\_27a}). (\forall V1a \in \\ (2^{(2^{A\_27a})}). (\forall V2x \in (2^{A\_27a}). ((p (ap (ap (c\_2Ebool\_2EIN \\ (2^{A\_27a})\ V2x)\ V1a)) \Rightarrow (p (ap (ap (c\_2Ebool\_2EIN\ (2^{A\_27a})\ V2x) \\ (ap (c\_2Emeasure\_2Esubsets\ A\_27a) (ap (ap (c\_2Emeasure\_2Esigma \\ A\_27a)\ V0sp)\ V1a)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in (ty\_2Epair\_2Eprod \\ (2^{A\_27a})\ (2^{(2^{A\_27a})})). ((p (ap (c\_2Emeasure\_2Esigma\_algebra \\ A\_27a)\ V0a)) \Leftrightarrow ((p (ap (ap (c\_2Emeasure\_2Esubset\_class\ A\_27a) \\ (ap (c\_2Emeasure\_2Espace\ A\_27a)\ V0a)) (ap (c\_2Emeasure\_2Esubsets \\ A\_27a)\ V0a))) \wedge ((p (ap (ap (c\_2Ebool\_2EIN\ (2^{A\_27a})\ (c\_2Epred\_set\_2EEMPTY \\ A\_27a)) (ap (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0a))) \wedge ((\forall V1s \in \\ (2^{A\_27a}). ((p (ap (ap (c\_2Ebool\_2EIN\ (2^{A\_27a})\ V1s) (ap (c\_2Emeasure\_2Esubsets \\ A\_27a)\ V0a))) \Rightarrow (p (ap (ap (c\_2Ebool\_2EIN\ (2^{A\_27a})\ (ap (ap (c\_2Epred\_set\_2EDIFF \\ A\_27a) (ap (c\_2Emeasure\_2Espace\ A\_27a)\ V0a))\ V1s)) (ap (c\_2Emeasure\_2Esubsets \\ A\_27a)\ V0a)))))) \wedge (\forall V2f \in ((2^{A\_27a})^{ty\_2Enum\_2Enum}). (( \\ p (ap (ap (c\_2Ebool\_2EIN\ ((2^{A\_27a})^{ty\_2Enum\_2Enum})\ V2f) (ap ( \\ ap (c\_2Epred\_set\_2EFUNSET\ ty\_2Enum\_2Enum\ (2^{A\_27a})\ (c\_2Epred\_set\_2EUNIV \\ ty\_2Enum\_2Enum)) (ap (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0a)))))) \Rightarrow \\ (p (ap (ap (c\_2Ebool\_2EIN\ (2^{A\_27a})\ (ap (c\_2Epred\_set\_2EBIGUNION \\ A\_27a) (ap (ap (c\_2Epred\_set\_2EIMAGE\ ty\_2Enum\_2Enum\ (2^{A\_27a}) \\ V2f) (c\_2Epred\_set\_2EUNIV\ ty\_2Enum\_2Enum)))))) (ap (c\_2Emeasure\_2Esubsets \\ A\_27a)\ V0a)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0x \in A\_27a. (\forall V1y \in A\_27b. (\forall V2a \in A\_27a. (\forall V3b \in \\ A\_27b. (((ap (ap (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap (ap \\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0f \in ((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}).(\forall V1v \in \\ & A\_27a.((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V1v)\ (ap\ (c\_2Epred\_set\_2EGSPEC \\ & \quad A\_27a\ A\_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A\_27b.((ap\ (ap\ (c\_2Epair\_2E\_2C \\ & \quad A\_27a\ 2)\ V1v)\ c\_2Ebool\_2ET) = (ap\ V0f\ V2x)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in \\ & (2^{A\_27a}).(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a) \\ & V2x)\ (ap\ (ap\ (c\_2Epred\_set\_2EINTER\ A\_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap \\ & (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\ & A\_27a)\ V2x)\ V1t)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in \\ & (2^{A\_27a}).(\forall V2u \in (2^{A\_27a}).((p\ (ap\ (ap\ (c\_2Epred\_set\_2ESUBSET \\ & A\_27a)\ V0s)\ (ap\ (ap\ (c\_2Epred\_set\_2EINTER\ A\_27a)\ V1t)\ V2u))) \Leftrightarrow \\ & ((p\ (ap\ (ap\ (c\_2Epred\_set\_2ESUBSET\ A\_27a)\ V0s)\ V1t)) \wedge (p\ (ap\ (ap \\ & (c\_2Epred\_set\_2ESUBSET\ A\_27a)\ V0s)\ V2u)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0a \in (2^{A\_27a}).(\forall V1b \in (2^{A\_27b}).(\forall V2c \in \\ & (2^{A\_27b}).((ap\ (ap\ (c\_2Epred\_set\_2EFUNSET\ A\_27a\ A\_27b)\ V0a) \\ & (ap\ (ap\ (c\_2Epred\_set\_2EINTER\ A\_27b)\ V1b)\ V2c)) = (ap\ (ap\ (c\_2Epred\_set\_2EINTER \\ & (A\_27b^{A\_27a})\ (ap\ (ap\ (c\_2Epred\_set\_2EFUNSET\ A\_27a\ A\_27b)\ V0a) \\ & V1b))\ (ap\ (ap\ (c\_2Epred\_set\_2EFUNSET\ A\_27a\ A\_27b)\ V0a)\ V2c)))))) \end{aligned} \quad (42)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (43)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (44)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (46)$$

Assume the following.

$$(\forall V0A \in 2.((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (47)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg( \\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ( \\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (51)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \end{aligned} \quad (52)$$



**Theorem 1**

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0sp \in (2^{A_{.27a}}). (\forall V1p \in \\
& (2^{(2^{A_{.27a}})}). (\forall V2a \in (2^{(2^{A_{.27a}})}). (((p\ (ap\ (ap\ (c\_2Emeasure\_2Esubset\_class \\
A_{.27a})\ V0sp)\ V1p)) \wedge ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (2^{A_{.27a}})\ (c\_2Epred\_set\_2EEMPTY \\
A_{.27a})\ V1p)) \wedge ((p\ (ap\ (ap\ (c\_2Epred\_set\_2ESUBSET\ (2^{A_{.27a}}) \\
V2a)\ V1p)) \wedge ((\forall V3s \in (2^{A_{.27a}}). ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\
(2^{A_{.27a}})\ V3s)\ (ap\ (ap\ (c\_2Epred\_set\_2EINTER\ (2^{A_{.27a}})\ V1p) \\
(ap\ (c\_2Emeasure\_2Esubsets\ A_{.27a})\ (ap\ (ap\ (c\_2Emeasure\_2Esigma \\
A_{.27a})\ V0sp)\ V2a)))))) \Rightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (2^{A_{.27a}})\ (ap \\
(ap\ (c\_2Epred\_set\_2EDIFF\ A_{.27a})\ V0sp)\ V3s))\ V1p)))) \wedge (\forall V4f \in \\
((2^{A_{.27a}})^{ty\_2Enum\_2Enum}). ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ ((2^{A_{.27a}})^{ty\_2Enum\_2Enum})) \\
V4f)\ (ap\ (ap\ (c\_2Epred\_set\_2EFUNSET\ ty\_2Enum\_2Enum\ (2^{A_{.27a}}) \\
(c\_2Epred\_set\_2EUNIV\ ty\_2Enum\_2Enum))\ (ap\ (ap\ (c\_2Epred\_set\_2EINTER \\
(2^{A_{.27a}})\ V1p)\ (ap\ (c\_2Emeasure\_2Esubsets\ A_{.27a})\ (ap\ (ap\ (c\_2Emeasure\_2Esigma \\
A_{.27a})\ V0sp)\ V2a)))))) \Rightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (2^{A_{.27a}})\ (ap \\
(c\_2Epred\_set\_2EBIGUNION\ A_{.27a})\ (ap\ (ap\ (c\_2Epred\_set\_2EIMAGE \\
ty\_2Enum\_2Enum\ (2^{A_{.27a}})\ V4f)\ (c\_2Epred\_set\_2EUNIV\ ty\_2Enum\_2Enum)))) \\
V1p)))))) \Rightarrow (p\ (ap\ (ap\ (c\_2Epred\_set\_2ESUBSET\ (2^{A_{.27a}})\ (ap \\
(c\_2Emeasure\_2Esubsets\ A_{.27a})\ (ap\ (ap\ (c\_2Emeasure\_2Esigma\ A_{.27a}) \\
V0sp)\ V2a)))\ V1p))))))
\end{aligned}$$