

thm_2Emeasure_2ESIGMA_PROPERTY_DISJOINT_LEMMA2 (TMddr4NqGpknyhBQam3H122szUvNmGU6nvT)

October 26, 2020

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (1)$$

Let $c_2Earithmetic_2Enum_CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Earithmetic_2Enum_CASE\ A_27a \in \left(((A_27a^{(A_27a^{*y} \rightarrow 2Enum_2Enum)})^{A_27a})^{ty_2Enum_2Enum} \right) \quad (2)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$.

Definition 3 We define $c_2Ebool_2EBOUNDED$ to be $(\lambda V0v \in 2.c_2Ebool_2ET)$.

Definition 4 We define $c_2Ecombin_2EK$ to be $\lambda A.\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$.

Definition 5 We define $c_2Ecombin_2ES$ to be $\lambda A.\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$.

Definition 6 We define $c_2Ecombin_2EI$ to be $\lambda A.\lambda A_27a : \iota.(ap\ (ap\ (c_2Ecombin_2ES\ A_27a\ (A_27a^{A_27a})\ A_27a))$.

Definition 7 We define c_2Ebool_2EIN to be $\lambda A.\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$.

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p \Rightarrow q)$ of type ι .

Definition 9 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a}))\ P))$.

Definition 10 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A.\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (ap\ (c_2Ebool_2E_21\ A_27a)\ s)\ t))$.

Definition 11 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A.\lambda A_27a : \iota.\lambda V0sp \in (2^{A_27a}).\lambda V1sts \in (2^{(2^{A_27a})})$.

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (3)$$

Let $c_2Emeasure_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Espace\ A_27a \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))}) \quad (4)$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (5)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{omega}) \quad (6)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (7)$$

Definition 12 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (8)$$

Definition 13 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Emeasure_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Esubsets\ A_27a \in ((2^{(2^{A_27a})})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))}) \quad (9)$$

Definition 14 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 15 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (10)$$

Definition 16 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b.(ap\ (c_2$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (11)$$

Definition 17 We define `c_2Epred_set_2EIMAGE` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Definition 18 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p \text{ (ap } P \ x)) \text{ then (the } (\lambda x.x \in A \wedge P \ x) \text{ of type } \iota \Rightarrow \iota).$

Definition 19 We define `c_2Ebool_2E_3F` to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(\text{ap } V0P \ (\text{ap } (c_2Emin_2E_40 \ A_27a))))$

Definition 20 We define `c_2Epred_set_2EBIGUNION` to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(\text{ap } (c_2Epred_set_2EIMAGE \ A_27a \ V0P))$

Definition 21 We define `c_2Ebool_2EF` to be $(\text{ap } (c_2Ebool_2E_21 \ 2) \ (\lambda V0t \in 2.V0t))$.

Definition 22 We define `c_2Epred_set_2EEMPTY` to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 23 We define `c_2Epred_set_2EINTER` to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(\text{ap } (c_2Epred_set_2EIMAGE \ A_27a \ V0s \ V1t))$

Definition 24 We define `c_2Epred_set_2EDISJOINT` to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(\text{ap } (c_2Epred_set_2EIMAGE \ A_27a \ V0s \ V1t))$

Definition 25 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2.(\text{ap } (\text{ap } c_2Emin_2E_3D_3D_3E \ V0t) \ c_2Ebool_2E_21))$

Definition 26 We define `c_2Epred_set_2EFUNSET` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A_27a}).\lambda V1Q \in (2^{A_27b}).(\text{ap } (c_2Epred_set_2EIMAGE \ A_27a \ V0P \ V1Q))$

Definition 27 We define `c_2Epred_set_2EDIFF` to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(\text{ap } (c_2Epred_set_2EIMAGE \ A_27a \ V0s \ V1t))$

Definition 28 We define `c_2Emeasure_2Eclosed_cdi` to be $\lambda A_27a : \iota.\lambda V0p \in (ty_2Epair_2Eprod \ (2^{A_27a}))$

Definition 29 We define `c_2Epred_set_2EBIGINTER` to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(\text{ap } (c_2Epred_set_2EIMAGE \ A_27a \ V0P))$

Definition 30 We define `c_2Emeasure_2Esmallest_closed_cdi` to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod \ (2^{A_27a}))$

Definition 31 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(\text{ap } (c_2Ebool_2E_21 \ 2) \ (\lambda V2t \in 2.V2t))))$

Definition 32 We define `c_2Epred_set_2EUNION` to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(\text{ap } (c_2Epred_set_2EIMAGE \ A_27a \ V0s \ V1t))$

Definition 33 We define `c_2Emeasure_2Ealgebra` to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod \ (2^{A_27a})) \ (2^{(2^{A_27a})})$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow ((\forall V0v \in A_27a.(\forall V1f \in \\ & (A_27a^{ty_2Eenum_2Eenum}).((\text{ap } (\text{ap } (\text{ap } (c_2Earithmetic_2Eenum_CASE \\ & A_27a) \ c_2Enum_2E0) \ V0v) \ V1f) = V0v))) \wedge (\forall V2n \in ty_2Eenum_2Eenum. \\ & (\forall V3v \in A_27a.(\forall V4f \in (A_27a^{ty_2Eenum_2Eenum}).((\text{ap } \\ & (\text{ap } (\text{ap } (c_2Earithmetic_2Eenum_CASE \ A_27a) \ (\text{ap } c_2Enum_2ESUC \\ & V2n)) \ V3v) \ V4f) = (\text{ap } V4f \ V2n))))))) \end{aligned} \quad (12)$$

Assume the following.

$$(\forall V0m \in ty_2Eenum_2Eenum.((V0m = c_2Enum_2E0) \vee (\exists V1n \in ty_2Eenum_2Eenum.(V0m = (\text{ap } c_2Enum_2ESUC \ V1n)))))) \quad (13)$$

Assume the following.

$$True \quad (14)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. ((p V0t1) \Rightarrow (p V1t2)) \Rightarrow ((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee \neg(p V0t))) \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge (p V1t2) \wedge (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow \neg(p V0t))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2. (\neg(p V0t) \Rightarrow ((p V0t) \Rightarrow False))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))) \quad (24)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (28)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((\neg(\forall V1x \in A_27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A_27a.(\neg(p (ap V0P V2x))))) \quad (29)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((\neg(\exists V1x \in A_27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A_27a.(\neg(p (ap V0P V2x))))) \quad (30)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in (2^{A_27a}).((\forall V2x \in A_27a.((p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((\forall V3x \in A_27a.(p (ap V0P V3x))) \wedge (\forall V4x \in A_27a.(p (ap V1Q V4x))))))) \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in 2.(((\forall V2x \in A_27a.(p (ap V0P V2x))) \wedge (p V1Q)) \Leftrightarrow (\forall V3x \in A_27a.((p (ap V0P V3x)) \wedge (p V1Q))))) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A_27a}).(((p V0P) \wedge (\forall V2x \in A_27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A_27a.((p V0P) \wedge (p (ap V1Q V3x))))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B)))))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B)))) \quad (43)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (44)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_{27} \in 2. (\forall V2y \in 2. (\forall V3y_{27} \in 2. (((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27})))))) \quad (45)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0P \in (2^{A_{27a}}). (\forall V1a \in A_{27a}. ((\exists V2x \in A_{27a}. ((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (46)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow \forall A_{27b}. \text{nonempty } A_{27b} \Rightarrow (\forall V0P \in ((2^{A_{27b}})^{A_{27a}}). ((\forall V1x \in A_{27a}. (\exists V2y \in A_{27b}. (p (ap (ap V0P V1x) V2y)))) \Leftrightarrow (\exists V3f \in (A_{27b}^{A_{27a}}). (\forall V4x \in A_{27a}. (p (ap (ap V0P V4x) (ap V3f V4x)))))))))) \quad (47)$$

Assume the following.

$$(\forall V0v \in 2. ((p (ap c_{2Ebool_2EBOUNDED} V0v)) \Leftrightarrow \text{True})) \quad (48)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0x \in A_{27a}. ((ap (c_{2Ecombin_2EI} A_{27a}) V0x) = V0x)) \quad (49)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0x \in (2^{A_{27a}}). (\forall V1y \in (2^{(2^{A_{27a}})}). ((ap (c_{2Emeasure_2Espace} A_{27a}) (ap (ap (c_{2Epair_2E_2C} (2^{A_{27a}}) (2^{(2^{A_{27a}})})) V0x) V1y)) = V0x)))) \quad (50)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0x \in (2^{A_{27a}}). (\forall V1y \in (2^{(2^{A_{27a}})}). ((ap (c_{2Emeasure_2Esubsets} A_{27a}) (ap (ap (c_{2Epair_2E_2C} (2^{A_{27a}}) (2^{(2^{A_{27a}})})) V0x) V1y)) = V1y)))) \quad (51)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a \in (ty_2Epair_2Eprod \\ & (2^{A.27a}) (2^{(2^{A.27a})})).((p (ap (c.2Emeasure_2Ealgebra\ A.27a) \\ & V0a)) \Rightarrow (p (ap (ap (c.2Ebool_2EIN (2^{A.27a})) (c.2Epred_set_2EEMPTY \\ & A.27a)) (ap (c.2Emeasure_2Esubsets\ A.27a) V0a)))))) \end{aligned} \quad (52)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a \in (ty_2Epair_2Eprod \\ & (2^{A.27a}) (2^{(2^{A.27a})})).((ap (c.2Emeasure_2Espace\ A.27a) (\\ & ap (c.2Emeasure_2Esmallest_closed_cdi\ A.27a) V0a)) = (ap (c.2Emeasure_2Espace \\ & A.27a) V0a))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a \in (ty_2Epair_2Eprod \\ & (2^{A.27a}) (2^{(2^{A.27a})})).((p (ap (c.2Emeasure_2Ealgebra\ A.27a) \\ & V0a)) \Rightarrow ((p (ap (ap (c.2Epred_set_2ESUBSET (2^{A.27a})) (ap (c.2Emeasure_2Esubsets \\ & A.27a) V0a)) (ap (c.2Emeasure_2Esubsets\ A.27a) (ap (c.2Emeasure_2Esmallest_closed_cdi \\ & A.27a) V0a)))) \wedge ((p (ap (c.2Emeasure_2Eclosed_cdi\ A.27a) (ap \\ & (c.2Emeasure_2Esmallest_closed_cdi\ A.27a) V0a))) \wedge (p (ap (\\ & ap (c.2Emeasure_2Esubset_class\ A.27a) (ap (c.2Emeasure_2Espace \\ & A.27a) V0a)) (ap (c.2Emeasure_2Esubsets\ A.27a) (ap (c.2Emeasure_2Esmallest_closed_cdi \\ & A.27a) V0a)))))))))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0p \in (ty_2Epair_2Eprod \\ & (2^{A.27a}) (2^{(2^{A.27a})})).(\forall V1s \in (2^{A.27a}).(\forall V2t \in \\ & (2^{A.27a}).(((p (ap (ap (c.2Ebool_2EIN (2^{A.27a})) (c.2Epred_set_2EEMPTY \\ & A.27a)) (ap (c.2Emeasure_2Esubsets\ A.27a) V0p))) \wedge ((p (ap (c.2Emeasure_2Eclosed_cdi \\ & A.27a) V0p)) \wedge ((p (ap (ap (c.2Ebool_2EIN (2^{A.27a})) V1s) (ap (c.2Emeasure_2Esubsets \\ & A.27a) V0p))) \wedge ((p (ap (ap (c.2Ebool_2EIN (2^{A.27a})) V2t) (ap (c.2Emeasure_2Esubsets \\ & A.27a) V0p))) \wedge (p (ap (ap (c.2Epred_set_2EDISJOINT\ A.27a) V1s) \\ & V2t)))))) \Rightarrow (p (ap (ap (c.2Ebool_2EIN (2^{A.27a})) (ap (ap (c.2Epred_set_2EUNION \\ & A.27a) V1s) V2t)) (ap (c.2Emeasure_2Esubsets\ A.27a) V0p))))))))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0p \in (ty_2Epair_2Eprod \\ & (2^{A.27a}) (2^{(2^{A.27a})})).(\forall V1s \in (2^{A.27a}).(((p (ap (c.2Emeasure_2Eclosed_cdi \\ & A.27a) V0p)) \wedge (p (ap (ap (c.2Ebool_2EIN (2^{A.27a})) V1s) (ap (c.2Emeasure_2Esubsets \\ & A.27a) V0p)))))) \Rightarrow (p (ap (ap (c.2Ebool_2EIN (2^{A.27a})) (ap (ap (c.2Epred_set_2EDIFF \\ & A.27a) (ap (c.2Emeasure_2Espace\ A.27a) V0p)) V1s)) (ap (c.2Emeasure_2Esubsets \\ & A.27a) V0p)))))) \end{aligned} \quad (56)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0p \in (ty_2Epair_2Eprod \\
& \quad (2^{A.27a}) (2^{(2^{A.27a})})). (\forall V1f \in ((2^{A.27a})^{ty_2Enum_2Enum}). \\
& \quad (((p (ap (c_2Emeasure_2Eclosed_cdi\ A.27a)\ V0p)) \wedge ((p (ap (ap (\\
& \quad c_2Ebool_2EIN ((2^{A.27a})^{ty_2Enum_2Enum}))\ V1f) (ap (ap (c_2Epred_set_2EFUNSET \\
& \quad ty_2Enum_2Enum (2^{A.27a})) (c_2Epred_set_2EUNIV\ ty_2Enum_2Enum))) \\
& \quad (ap (c_2Emeasure_2Esubsets\ A.27a)\ V0p)))) \wedge (\forall V2m \in ty_2Enum_2Enum. \\
& \quad (\forall V3n \in ty_2Enum_2Enum. ((\neg(V2m = V3n)) \Rightarrow (p (ap (ap (c_2Epred_set_2EDISJOINT \\
& \quad A.27a) (ap\ V1f\ V2m)) (ap\ V1f\ V3n)))))) \Rightarrow (p (ap (ap (c_2Ebool_2EIN \\
& \quad (2^{A.27a})) (ap (c_2Epred_set_2EBIGUNION\ A.27a) (ap (ap (c_2Epred_set_2EIMAGE \\
& \quad ty_2Enum_2Enum (2^{A.27a}))\ V1f) (c_2Epred_set_2EUNIV\ ty_2Enum_2Enum)))) \\
& \quad (ap (c_2Emeasure_2Esubsets\ A.27a)\ V0p))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0p \in (ty_2Epair_2Eprod \\
& \quad (2^{A.27a}) (2^{(2^{A.27a})})). (\forall V1f \in ((2^{A.27a})^{ty_2Enum_2Enum}). \\
& \quad (((p (ap (c_2Emeasure_2Eclosed_cdi\ A.27a)\ V0p)) \wedge ((p (ap (ap (\\
& \quad c_2Ebool_2EIN ((2^{A.27a})^{ty_2Enum_2Enum}))\ V1f) (ap (ap (c_2Epred_set_2EFUNSET \\
& \quad ty_2Enum_2Enum (2^{A.27a})) (c_2Epred_set_2EUNIV\ ty_2Enum_2Enum))) \\
& \quad (ap (c_2Emeasure_2Esubsets\ A.27a)\ V0p)))) \wedge (((ap\ V1f\ c_2Enum_2EO) = \\
& \quad (c_2Epred_set_2EEMPTY\ A.27a)) \wedge (\forall V2n \in ty_2Enum_2Enum. \\
& \quad (p (ap (ap (c_2Epred_set_2ESUBSET\ A.27a) (ap\ V1f\ V2n)) (ap\ V1f\ (\\
& \quad ap\ c_2Enum_2ESUC\ V2n)))))) \Rightarrow (p (ap (ap (c_2Ebool_2EIN (2^{A.27a})) \\
& \quad (ap (c_2Epred_set_2EBIGUNION\ A.27a) (ap (ap (c_2Epred_set_2EIMAGE \\
& \quad ty_2Enum_2Enum (2^{A.27a}))\ V1f) (c_2Epred_set_2EUNIV\ ty_2Enum_2Enum)))) \\
& \quad (ap (c_2Emeasure_2Esubsets\ A.27a)\ V0p))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a \in (ty_2Epair_2Eprod \\
& \quad (2^{A.27a}) (2^{(2^{A.27a})})). ((p (ap (c_2Emeasure_2Ealgebra\ A.27a) \\
& \quad V0a)) \Rightarrow (\forall V1s \in (2^{A.27a}). (\forall V2t \in (2^{A.27a}). (((p (\\
& \quad ap (ap (c_2Ebool_2EIN (2^{A.27a}))\ V1s) (ap (c_2Emeasure_2Esubsets \\
& \quad A.27a)\ V0a))) \wedge (p (ap (ap (c_2Ebool_2EIN (2^{A.27a}))\ V2t) (ap (c_2Emeasure_2Esubsets \\
& \quad A.27a) (ap (c_2Emeasure_2Esmallest_closed_cdi\ A.27a)\ V0a)))))) \Rightarrow \\
& \quad (p (ap (ap (c_2Ebool_2EIN (2^{A.27a})) (ap (ap (c_2Epred_set_2EINTER \\
& \quad A.27a)\ V1s)\ V2t)) (ap (c_2Emeasure_2Esubsets\ A.27a) (ap (c_2Emeasure_2Esmallest_closed_cdi \\
& \quad A.27a)\ V0a))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0x \in A.27a. (\forall V1y \in A.27b. (\forall V2a \in A.27a. (\forall V3b \in \\
& \quad A.27b. (((ap (ap (c_2Epair_2E_2C\ A.27a\ A.27b)\ V0x)\ V1y) = (ap (ap \\
& \quad (c_2Epair_2E_2C\ A.27a\ A.27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & (2^{A_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t))))))) \end{aligned} \quad (61)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}). (\forall V1v \in \\ & A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC \\ & A_27a\ A_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A_27b. ((ap\ (ap\ (c_2Epair_2E_2C \\ & A_27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x)))))) \end{aligned} \quad (62)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\neg (p\ (ap\ (ap \\ & (c_2Ebool_2EIN\ A_27a)\ V0x)\ (c_2Epred_set_2EEMPTY\ A_27a)))))) \end{aligned} \quad (63)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27a)\ V0x)\ (c_2Epred_set_2EUNIV\ A_27a)))))) \end{aligned} \quad (64)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (p\ (ap\ (\\ & ap\ (c_2Epred_set_2ESUBSET\ A_27a)\ (c_2Epred_set_2EEMPTY\ A_27a)) \\ & V0s)))) \end{aligned} \quad (65)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & (2^{A_27a}). (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\ & V2x)\ (ap\ (ap\ (c_2Epred_set_2EINTER\ A_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap \\ & (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27a)\ V2x)\ V1t))))))) \end{aligned} \quad (66)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & (2^{A_27a}). ((ap\ (ap\ (c_2Epred_set_2EINTER\ A_27a)\ V0s)\ V1t) = (\\ & ap\ (ap\ (c_2Epred_set_2EINTER\ A_27a)\ V1t)\ V0s)))))) \end{aligned} \quad (67)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & (2^{A_27a}). (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\ & V2x)\ (ap\ (ap\ (c_2Epred_set_2EDIFF\ A_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (\\ & ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \wedge (\neg (p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27a)\ V2x)\ V1t))))))) \end{aligned} \quad (68)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0y \in A_27b. (\forall V1s \in (2^{A_27a}). (\forall V2f \in (A_27b^{A_27a}). \\
& \quad ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V0y)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\
& \quad A_27a\ A_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A_27a. ((V0y = (ap\ V2f\ V3x)) \wedge \\
& \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1s))))))
\end{aligned} \tag{69}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in (A_27b^{A_27a}). (\forall V1P \in (2^{A_27a}). (\forall V2Q \in \\
& \quad (2^{A_27b}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (A_27b^{A_27a}))\ V0f)\ (ap\ (ap \\
& \quad (c_2Epred_set_2EFUNSET\ A_27a\ A_27b)\ V1P)\ V2Q))) \Leftrightarrow (\forall V3x \in \\
& \quad A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1P)) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad A_27b)\ (ap\ V0f\ V3x))\ V2Q))))))
\end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1sos \in \\
& \quad (2^{(2^{A_27a})}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ (ap\ (c_2Epred_set_2EBIGUNION \\
& \quad A_27a)\ V1sos))) \Leftrightarrow (\exists V2s \in (2^{A_27a}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad A_27a)\ V0x)\ V2s)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A_27a}))\ V2s)\ V1sos))))))
\end{aligned} \tag{71}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1B \in \\
& \quad (2^{(2^{A_27a})}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ (ap\ (c_2Epred_set_2EBIGINTER \\
& \quad A_27a)\ V1B))) \Leftrightarrow (\forall V2P \in (2^{A_27a}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad (2^{A_27a}))\ V2P)\ V1B)) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ V2P))))))
\end{aligned} \tag{72}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{73}$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{74}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \tag{75}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \tag{76}$$

Assume the following.

$$(\forall V0A \in 2.((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (77)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(\\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (78)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (79)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (80)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (81)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \end{aligned} \quad (82)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (83)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (84)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (85)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (86)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (87)$$

Theorem 1

$$\begin{aligned} & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0a \in (\text{ty_2Epair_2Eprod} \\ & (2^{A_{27a}}) (2^{(2^{A_{27a}})})). ((p (ap (c_2Emeasure_2Ealgebra } A_{27a} \\ & V0a))) \Rightarrow (\forall V1s \in (2^{A_{27a}}). (\forall V2t \in (2^{A_{27a}}). (((p (\\ & ap (ap (c_2Ebool_2EIN (2^{A_{27a}})) V1s) (ap (c_2Emeasure_2Esubsets \\ & A_{27a}) (ap (c_2Emeasure_2Esmallest_closed_cdi } A_{27a}) V0a)))) \wedge \\ & (p (ap (ap (c_2Ebool_2EIN (2^{A_{27a}})) V2t) (ap (c_2Emeasure_2Esubsets \\ & A_{27a}) (ap (c_2Emeasure_2Esmallest_closed_cdi } A_{27a}) V0a)))) \Rightarrow \\ & (p (ap (ap (c_2Ebool_2EIN (2^{A_{27a}})) (ap (ap (c_2Epred_set_2EINTER \\ & A_{27a}) V1s) V2t)) (ap (c_2Emeasure_2Esubsets } A_{27a}) (ap (c_2Emeasure_2Esmallest_closed_cdi \\ & A_{27a}) V0a))))))))) \end{aligned}$$