

thm_2Emeasure_2ESIGMA__REDUCE

(TMMzZuU96FScabEJ4xiKh4Srjce7PsMVjzg)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2))) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x)$

Definition 3 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 4 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P (\text{ap } (\text{c_2Emin_2E_40 } A))))$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty_2Epair_2Eprod } A0 A1) \quad (1)$$

Let `c_2Emeasure_2Esubsets` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \text{c_2Emeasure_2Esubsets } A. 27a \in (2^{(2^{A-27a})}) (\text{ty_2Epair_2Eprod } (2^{A-27a}) (2^{(2^{A-27a})})) \quad (2)$$

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 7 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t))))$

Let `c_2Epair_2EABS__prod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \forall A. 27b. \text{nonempty } A. 27b \Rightarrow \text{c_2Epair_2EABS_prod } A. 27a A. 27b \in ((\text{ty_2Epair_2Eprod } A. 27a A. 27b))^{((2^{A-27b})^{A-27a})} \quad (3)$$

Definition 8 We define `c_2Epair_2E_2C` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0x \in A. 27a. \lambda V1y \in A. 27b. (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a})))$

Definition 9 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A-27a}). (ap\ V1f\ V0x)))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A-27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A-27b}}) \quad (4)$$

Definition 10 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^{A-27a})}). (ap\ (c_2Epred_set_2EGSPEC\ A_27a\ P))$

Definition 11 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in (2^{A-27a}). (ap\ (c_2Ebool_2E21\ 2)\ V2t)\ V0s)))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (5)$$

Definition 12 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2ET)$.

Definition 13 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A-27a}). \lambda V1s \in (2^{A-27a}). (ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in (2^{A-27a}). (ap\ (c_2Ebool_2E21\ 2)\ V2t)\ V0s)))$

Definition 14 We define $c_2Epred_set_2Ecountable$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A-27a}). (ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in (2^{A-27a}). (ap\ (c_2Ebool_2E21\ 2)\ V2t)\ V0s)))$

Definition 15 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ V2t)\ V0t1))))$

Definition 16 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in (2^{A-27a}). (ap\ (c_2Ebool_2E21\ 2)\ V2t)\ V0s)))$

Let $c_2Emeasure_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Emeasure_2Espace\ A_27a \in ((2^{A-27a})^{(ty_2Epair_2Eprod\ (2^{A-27a})\ (2^{(2^{A-27a})}))}) \quad (6)$$

Definition 17 We define $c_2Ebool_2E_F$ to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2. V0t))$.

Definition 18 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_21\ 2)\ V0t))$

Definition 19 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in (2^{A-27a}). (ap\ (c_2Ebool_2E21\ 2)\ V2t)\ V0s)))$

Definition 20 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2E_F)$.

Definition 21 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A_27a : \iota. \lambda V0sp \in (2^{A-27a}). \lambda V1sts \in (2^{(2^{A-27a})}). (ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in (2^{A-27a}). (ap\ (c_2Ebool_2E21\ 2)\ V2t)\ V0sp)))$

Definition 22 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A_27a : \iota. \lambda V0a \in (ty_2Epair_2Eprod\ (2^{A-27a})\ (2^{(2^{A-27a})})). (ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in (2^{A-27a}). (ap\ (c_2Ebool_2E21\ 2)\ V2t)\ V0a)))$

Definition 23 We define $c_2Emeasure_2Esigma_algebra$ to be $\lambda A_27a : \iota. \lambda V0a \in (ty_2Epair_2Eprod\ (2^{A-27a})\ (2^{(2^{A-27a})})). (ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in (2^{A-27a}). (ap\ (c_2Ebool_2E21\ 2)\ V2t)\ V0a)))$

Definition 24 We define $c_2Epred_set_2EBIGINTER$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^{A-27a})}). (ap\ (c_2Epred_set_2EBIGUNION\ A_27a\ P))$

Definition 25 We define $c_2Emeasure_2Esigma$ to be $\lambda A_27a : \iota. \lambda V0sp \in (2^{A-27a}). \lambda V1st \in (2^{(2^{A-27a})}). (ap\ (c_2Emeasure_2Esubset_class\ A_27a\ sp))$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee \neg(p V0t))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow \neg(p V0t))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2. (\neg(p V0t) \Rightarrow ((p V0t) \Rightarrow False))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (15)$$

Assume the following.

$$((\forall V0t \in 2. (\neg(\neg(p V0t)) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (16)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a. ((V0x = V0x) \Leftrightarrow True)) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (18)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A-27a}).(\neg(\forall V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p (ap V0P V2x))))) \quad (19)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C))))) \quad (20)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (21)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge (((\neg(p V0A) \vee \neg(p V1B)) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B))))) \quad (22)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0a \in (ty_2Epair_2Eprod (2^{A-27a}) (2^{(2^{A-27a})})).((ap (ap (c_2Epair_2E_2C (2^{A-27a}) (2^{(2^{A-27a})}))) (ap (c_2Emeasure_2Espace A.27a) V0a)) (ap (c_2Emeasure_2Esubsets A.27a) V0a)) = V0a)) \quad (23)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0sp \in (2^{A-27a}).(\forall V1a \in (2^{(2^{A-27a})}).((ap (c_2Emeasure_2Espace A.27a) (ap (ap (c_2Emeasure_2Esigma A.27a) V0sp) V1a)) = V0sp))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.(\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \quad (25)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (27)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow (p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(\\ & p V2r)) \vee (\neg(p V1q)))))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (30) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (31) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (32) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (33) \end{aligned}$$

Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0sp \in (2^{A-27a}).(\forall V1a \in \\ & (2^{(2^{A-27a})}).((ap (ap (c.2Epair_2E_2C (2^{A-27a}) (2^{(2^{A-27a})})) \\ & V0sp) (ap (c.2Emeasure_2Esubsets A_27a) (ap (ap (c.2Emeasure_2Esigma \\ & A_27a) V0sp) V1a))) = (ap (ap (c.2Emeasure_2Esigma A_27a) V0sp) \\ & V1a)))) \end{aligned}$$