

thm_2Emeasure_2Efinite__additivity__sufficient__for__finite__spaces2 (TMKWDfocLzcr33SS6VNxG9L7fxioBwQodT2)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2E$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_2E$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2E$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Emeasure_2Emeasure : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Emeasure\ A_27a \in ((ty_2Erealax_2Ereal^{(2^{A-27a})})(ty_2Epair_2Eprod\ (2^{A-27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A-27a})}))\ (ty_2Erealax_2Ereal^{(2^{A-27a})}))) \tag{3}$$

Let $c_2Emeasure_2Emeasurable_sets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Emeasurable_sets\ A_27a \in (((2^{(2^{A-27a})})(ty_2Epair_2Eprod\ (2^{A-27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A-27a})}))\ (ty_2Erealax_2Ereal^{(2^{A-27a})})))) \tag{4}$$

Let $c_2Emeasure_2Em_space : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Em_space\ A_27a \in ((2^{A_27a})(ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal(2^{A_27a})))))) \quad (5)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (6)$$

Definition 7 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 8 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2))))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{(2^{A_27b})^{A_27a}})}) \quad (7)$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2Epair_2EABS_prod\ x\ y))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (8)$$

Definition 11 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in A_27a. (ap\ V0f\ s)$

Definition 12 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x. x \in A \wedge p\ (ap\ P\ x)) \text{ of type } \iota \Rightarrow \iota)$.

Definition 13 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40\ P))))$

Definition 14 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^{A_27a})}). (ap\ (c_2Epred_set_2EGSPEC\ P))$

Definition 15 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b^{A_27c}). \lambda V1g \in (A_27a^{A_27c}). (ap\ V0f\ V1g)$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (10)$$

Definition 16 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Esum : \iota$ be given. Assume the following.

$$c_2Ereal_2Esum \in ((ty_2Erealax_2Ereal^{(ty_2Erealax_2Ereal^{ty_2Enum_2Enum})})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \quad (11)$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (12)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (13)$$

Definition 17 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 18 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 19 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 20 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 21 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (14)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (15)$$

Definition 22 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ (t$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal}) \quad (16)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (17)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})}) \quad (18)$$

Definition 23 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 24 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Edist\ A_27a \in ((ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})) \quad (26)$$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (27)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopology\ A_27a \in ((ty_2Etopology_2Etopology\ A_27a)^{(2^{(2^{A_27a})})}) \quad (28)$$

Definition 33 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Emetric_2Emetric\ A_27a).(ap$

Let $c_2Enets_2Eextends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Enets_2Eextends\ A_27a\ A_27b \in (((2^{(ty_2Epair_2Eprod\ (ty_2Etopology_2Etopology\ A_27a)\ (2^{A_27b})^{A_27b})}))_{A_27a})_{(A_27a)^{A_27b}} \quad (29)$$

Definition 34 We define $c_2Eseq_2E_2D_2D_3E$ to be $\lambda V0x \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).\lambda V1x$

Definition 35 We define c_2Eseq_2Esums to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).\lambda V1s \in ty_2$

Definition 36 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 37 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2$

Definition 38 We define $c_2Epred_set_2EDISJOINT$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2$

Definition 39 We define $c_2Epred_set_2EFUNSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A_27a}).\lambda V1Q \in (2^{A_27b}).(ap\ (c_2$

Definition 40 We define $c_2Emeasure_2Ecountably_additive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2$

Definition 41 We define $c_2Emeasure_2Epositive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2$

Let $c_2Emeasure_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Esubsets\ A_27a \in ((2^{(2^{A_27a})})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})})})) \quad (30)$$

Definition 42 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2$

Definition 43 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b)^{A_27a}.\lambda V1s \in (A_27a)^{A_27b}$

Definition 44 We define $c_2Epred_set_2Ecountable$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap\ (c_2Ebool_2E_3F$

Definition 45 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2$

Let $c_2Emeasure_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Espace\ A_27a \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})})})) \quad (31)$$

Definition 46 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2E$

Definition 47 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A_27a}).\lambda V1sts \in (2^{(2^{A_27a})})$

Definition 48 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))$

Definition 49 We define $c_2Emeasure_2Esigma_algebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))$

Definition 50 We define $c_2Emeasure_2Emeasure_space$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))$

Definition 51 We define $c_2Emeasure_2Eadditive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))$

Definition 52 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap\ (c_2E$

Definition 53 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap\ (c_2Ebool_2E21\ (2^{A_27a}))$

Assume the following.

$$True \quad (32)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (\\ & p\ V0t)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in \\ & 2.(((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in (2^{A.27a}).(\forall V1y \in \\ & (2^{(2^{A.27a})}).((ap\ (c.2Emeasure.2Espace\ A.27a)\ (ap\ (ap\ (c.2Epair.2E.2C \\ & (2^{A.27a})\ (2^{(2^{A.27a})}))\ V0x)\ V1y)) = V0x))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in (2^{A.27a}).(\forall V1y \in \\ & (2^{(2^{A.27a})}).((ap\ (c.2Emeasure.2Esubsets\ A.27a)\ (ap\ (ap\ (c.2Epair.2E.2C \\ & (2^{A.27a})\ (2^{(2^{A.27a})}))\ V0x)\ V1y)) = V1y))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in (ty.2Epair.2Eprod \\ & (2^{A.27a})\ (ty.2Epair.2Eprod\ (2^{(2^{A.27a})})\ (ty.2Erealax.2Ereal^{(2^{A.27a})}))). \\ & ((ap\ (ap\ (c.2Epair.2E.2C\ (2^{A.27a})\ (ty.2Epair.2Eprod\ (2^{(2^{A.27a})}) \\ & (ty.2Erealax.2Ereal^{(2^{A.27a})}))))\ (ap\ (c.2Emeasure.2Em_space \\ & A.27a)\ V0m))\ (ap\ (ap\ (c.2Epair.2E.2C\ (2^{(2^{A.27a})})\ (ty.2Erealax.2Ereal^{(2^{A.27a})}))) \\ & (ap\ (c.2Emeasure.2Emeasurable_sets\ A.27a)\ V0m))\ (ap\ (c.2Emeasure.2Emeasure \\ & A.27a)\ V0m))) = V0m)) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (ty.2Epair.2Eprod \\ & (2^{A.27a})\ (2^{(2^{A.27a})})).(\forall V1m \in (ty.2Erealax.2Ereal^{(2^{A.27a})}). \\ & (((p\ (ap\ (c.2Emeasure.2Esigma_algebra\ A.27a)\ V0s)) \wedge ((p\ (ap\ (\\ & c.2Epred_set.2EFINITE\ A.27a)\ (ap\ (c.2Emeasure.2Espace\ A.27a) \\ & V0s))) \wedge ((p\ (ap\ (c.2Emeasure.2Epositive\ A.27a)\ (ap\ (ap\ (c.2Epair.2E.2C \\ & (2^{A.27a})\ (ty.2Epair.2Eprod\ (2^{(2^{A.27a})})\ (ty.2Erealax.2Ereal^{(2^{A.27a})}))) \\ & (ap\ (c.2Emeasure.2Espace\ A.27a)\ V0s))\ (ap\ (ap\ (c.2Epair.2E.2C \\ & (2^{(2^{A.27a})})\ (ty.2Erealax.2Ereal^{(2^{A.27a})})))\ (ap\ (c.2Emeasure.2Esubsets \\ & A.27a)\ V0s))\ V1m)))) \wedge (p\ (ap\ (c.2Emeasure.2Eadditive\ A.27a)\ (ap \\ & (ap\ (c.2Epair.2E.2C\ (2^{A.27a})\ (ty.2Epair.2Eprod\ (2^{(2^{A.27a})}) \\ & (ty.2Erealax.2Ereal^{(2^{A.27a})}))))\ (ap\ (c.2Emeasure.2Espace\ A.27a) \\ & V0s))\ (ap\ (ap\ (c.2Epair.2E.2C\ (2^{(2^{A.27a})})\ (ty.2Erealax.2Ereal^{(2^{A.27a})}))) \\ & (ap\ (c.2Emeasure.2Esubsets\ A.27a)\ V0s))\ V1m)))))) \Rightarrow (p\ (ap\ (c.2Emeasure.2Emeasure_space \\ & A.27a)\ (ap\ (ap\ (c.2Epair.2E.2C\ (2^{A.27a})\ (ty.2Epair.2Eprod\ (2^{(2^{A.27a})}) \\ & (ty.2Erealax.2Ereal^{(2^{A.27a})})))\ (ap\ (c.2Emeasure.2Espace\ A.27a) \\ & V0s))\ (ap\ (ap\ (c.2Epair.2E.2C\ (2^{(2^{A.27a})})\ (ty.2Erealax.2Ereal^{(2^{A.27a})}))) \\ & (ap\ (c.2Emeasure.2Esubsets\ A.27a)\ V0s))\ V1m)))))) \end{aligned} \quad (41)$$

Theorem 1

$$\begin{aligned} & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0m \in (\text{ty_2Epair_2Eprod} \\ & (2^{A_{27a}}) (\text{ty_2Epair_2Eprod } (2^{(2^{A_{27a}})}) (\text{ty_2Erealax_2Ereal}^{(2^{A_{27a}})}))))). \\ & (((p (\text{ap } (\text{c_2Emeasure_2Esigma_algebra } A_{27a}) (\text{ap } (\text{ap } (\text{c_2Epair_2E_2C} \\ & (2^{A_{27a}}) (2^{(2^{A_{27a}})})) (\text{ap } (\text{c_2Emeasure_2Em_space } A_{27a}) \\ & V0m)) (\text{ap } (\text{c_2Emeasure_2Emeasurable_sets } A_{27a}) V0m)))))) \wedge ((\\ & p (\text{ap } (\text{c_2Epred_set_2EFINITE } A_{27a}) (\text{ap } (\text{c_2Emeasure_2Em_space} \\ & A_{27a}) V0m)))) \wedge ((p (\text{ap } (\text{c_2Emeasure_2Epositive } A_{27a}) V0m)) \wedge (\\ & p (\text{ap } (\text{c_2Emeasure_2Eadditive } A_{27a}) V0m)))))) \Rightarrow (p (\text{ap } (\text{c_2Emeasure_2Emeasure_space} \\ & A_{27a}) V0m)))) \end{aligned}$$