thm_2Emergesort_2Esort3__perm (TMGFnH8wqXuSLyFshCKGzRVNZr8TD3daxrp)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27a}$

Definition 5 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 6 *We define* c_2Ebool_2EF *to be* $(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \text{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \text{c\_2Ebool\_2EF}$

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\textbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \textbf{then}\ (the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{1}$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{2}$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{3}$$

Definition 11 *We define* c_2Emergesort_2Esort3 *to be* $\lambda A\_27a:\iota.\lambda V0R\in((2^{A\_27a})^{A\_27a}).\lambda V1x\in A\_27a.\lambda V$

Let $c\_2Elist\_2EAPPEND:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Elist\_2EAPPEND\ A\_27a\in(((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{4}$$

Let $c\_2Elist\_2EFILTER:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Elist\_2EFILTER\ A\_27a\in(((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27a})}) \tag{5}$$

Definition 12 *We define* c_2Esorting_2EPERM *to be* $\lambda A\_27a:\iota.\lambda V0L1\in(ty\_2Elist\_2Elist\ A\_27a).\lambda V1L2$

Assume the following.

$$True \tag{6}$$

Assume the following.

$$(\forall V0t1\in 2.(\forall V1t2\in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\ V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \tag{7}$$

Assume the following.

$$(\forall V0t\in 2.(False\Rightarrow(p\ V0t))) \tag{8}$$

Assume the following.

$$(\forall V0t\in 2.((p\ V0t)\vee(\neg(p\ V0t)))) \tag{9}$$

Assume the following.

$$(\forall V0t\in 2.(((True\wedge(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\wedge True)\Leftrightarrow(p\ V0t))\wedge(((False\wedge(p\ V0t))\Leftrightarrow False)\wedge((((p\ V0t)\wedge False)\Leftrightarrow False)\wedge(((p\ V0t)\wedge(p\ V0t))\Leftrightarrow(p\ V0t))))))) \tag{10}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x\in A\_27a.((V0x=V0x)\Leftrightarrow True)) \tag{11}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x\in A\_27a.(\forall V1y\in A\_27a.((V0x=V1y)\Leftrightarrow(V1y=V0x)))) \tag{12}$$

Assume the following.

$$(\forall V0t\in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow(p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(p\ V0t))))))) \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2) = V1t2)))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a))\ V0l) = V0l)) \wedge (\forall V1l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2l2 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V3h \in A\_27a.((ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3h)\ V1l1))\ V2l2) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3h)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V1l1)\ V2l2))))))) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a0 \in A\_27a.(\forall V1a1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2a0\_27 \in A\_27a.(\forall V3a1\_27 \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0a0)\ V1a1) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2a0\_27)\ V3a1\_27)) \Leftrightarrow ((V0a0 = V2a0\_27) \wedge (V1a1 = V3a1\_27))))))) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1l2 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2l1 \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0x)\ V2l1))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0x)\ V1l2))) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V2l1)\ V1l2)))))) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0L \in (ty\_2Elist\_2Elist\ A\_27a).(((p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V0L)\ (c\_2Elist\_2ENIL\ A\_27a))) \Leftrightarrow (V0L = (c\_2Elist\_2ENIL\ A\_27a))) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a))\ V0L)) \Leftrightarrow (V0L = (c\_2Elist\_2ENIL\ A\_27a))))) \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0L \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1x \in A\_27a.(((p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V0L)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V1x)\ (c\_2Elist\_2ENIL\ A\_27a)))) \Leftrightarrow (V0L = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V1x)\ (c\_2Elist\_2ENIL\ A\_27a)))) \land ((p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V1x)\ (c\_2Elist\_2ENIL\ A\_27a)))\ V0L)) \Leftrightarrow (V0L = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V1x)\ (c\_2Elist\_2ENIL\ A\_27a)))))))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1y \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V0x)\ V1y)) \Leftrightarrow ((ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V0x) = (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V1y))))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0y \in A\_27a.(\forall V1l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2x \in A\_27a.(\forall V3l2 \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V1l1) = (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2x)\ V3l2))) \Rightarrow ((ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0y)\ V1l1)) = (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2x)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0y)\ V3l2))))))))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2x \in A\_27a.(\forall V3l2 \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V1l1) = (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2x)\ V3l2))) \Rightarrow ((ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0l)\ V1l1)) = (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2x)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0l)\ V3l2))))))))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2l2 \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V1l1) = (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V2l2)) \Rightarrow ((ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0x)\ V1l1)) = (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0x)\ V2l2))))))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2l1\_27 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V3l2 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V4l2\_27 \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V1l1)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0l)\ V2l1\_27))) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V3l2)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0l)\ V4l2\_27))) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V1l1)\ V3l2)) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V2l1\_27)\ V4l2\_27)))))))))) \tag{24}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1x \in A\_27a.(\forall V2y \in A\_27a.(\forall V3z \in A\_27a.(p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V1x)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2y)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3z)\ (c\_2Elist\_2ENIL\ A\_27a)))))\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Emergesort\_2Esort3}\ A\_27a)\ V0R)\ V1x)\ V2y)\ V3z)))))))$$