

thm_2Emergesort_2Estable__cong
(TMa6H4im6zjYae8yUd7iSLtPSD9N7Q8b1Ex)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_21` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2))) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x)$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_27a}))))$

Definition 4 We define `c_2Ebool_2E_21` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V0t \in 2. V0t)$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t)) (\text{c_2Ebool_2E_21 } 2)))$

Let `ty_2Elist_2Elist` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_2Elist_2Elist } A0) \quad (1)$$

Let `c_2Elist_2Eappend` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow \text{c_2Elist_2Eappend } A_27a \in (((\text{ty_2Elist_2Elist } A_27a) (\text{ty_2Elist_2Elist } A_27a)) (\text{ty_2Elist_2Elist } A_27a)) \quad (2)$$

Let `c_2Elist_2Efilter` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow \text{c_2Elist_2Efilter } A_27a \in (((\text{ty_2Elist_2Elist } A_27a) (\text{ty_2Elist_2Elist } A_27a)) (2^{A_27a})) \quad (3)$$

Definition 7 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V2t \in 2. V2t)))$

Definition 8 We define `c_2Emergesort_2Estable` to be $\lambda A_27a : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1l1 \in (\text{ty_2Elist_2Elist } A_27a)$

Assume the following.

$$True \quad (4)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a.(p\ V0t) \Leftrightarrow (p\ V0t)))) \quad (5)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (6) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (7) \end{aligned}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (8)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg \\ & p\ V0t)))))) \quad (9) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (10) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\ & 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \quad (11) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0l1 \in (ty_2Elist_2Elist \\ & A_27a). (\forall V1l2 \in (ty_2Elist_2Elist\ A_27a). (\forall V2l3 \in \\ & (ty_2Elist_2Elist\ A_27a). (((ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a) \\ & V0l1)\ V1l2) = (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V0l1)\ V2l3)) \Leftrightarrow (V1l2 = \\ & V2l3)))))) \wedge (\forall V3l1 \in (ty_2Elist_2Elist\ A_27a). (\forall V4l2 \in \\ & (ty_2Elist_2Elist\ A_27a). (\forall V5l3 \in (ty_2Elist_2Elist\ A_27a). \\ & (((ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V4l2)\ V3l1) = (ap\ (ap\ (c_2Elist_2EAPPEND \\ & A_27a)\ V5l3)\ V3l1)) \Leftrightarrow (V4l2 = V5l3)))))) \quad (12) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1L \in \\
& \quad (ty_2Elist_2Elist\ A_27a). (\forall V2M \in (ty_2Elist_2Elist\ A_27a). \\
& \quad ((ap\ (ap\ (c_2Elist_2EFILTER\ A_27a)\ V0P)\ (ap\ (ap\ (c_2Elist_2EAPPEND \\
& \quad A_27a)\ V1L)\ V2M))) = (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ (ap\ (ap\ (c_2Elist_2EFILTER \\
& \quad A_27a)\ V0P)\ V1L))\ (ap\ (ap\ (c_2Elist_2EFILTER\ A_27a)\ V0P)\ V2M)))))) \\
& \hspace{15em} (13)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}). \\
& \quad (\forall V1l1 \in (ty_2Elist_2Elist\ A_27a). (\forall V2l2 \in (ty_2Elist_2Elist \\
& \quad A_27a). (\forall V3l3 \in (ty_2Elist_2Elist\ A_27a). (\forall V4l4 \in \\
& \quad (ty_2Elist_2Elist\ A_27a). ((p\ (ap\ (ap\ (ap\ (c_2Emergesort_2Estable \\
& \quad A_27a)\ V0R)\ V1l1)\ V2l2)) \wedge (p\ (ap\ (ap\ (ap\ (c_2Emergesort_2Estable \\
& \quad A_27a)\ V0R)\ V3l3)\ V4l4)))) \Rightarrow (p\ (ap\ (ap\ (ap\ (c_2Emergesort_2Estable \\
& \quad A_27a)\ V0R)\ (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V1l1)\ V3l3))\ (ap\ (\\
& \quad ap\ (c_2Elist_2EAPPEND\ A_27a)\ V2l2)\ V4l4)))))))))
\end{aligned}$$