

thm_2Emetric_2EISMET__R1 (TMdPftxFoaKcq8W4ze4RiChE6G3BaRSpEQg)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty ty_2Ehreal_2Ehreal \tag{3}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty ty_2Erealax_2Ereal \tag{4}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)} ty_2Erealax_2Ereal) \tag{5}$$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 10 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (t$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)))(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal) \quad (6)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (7)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)} \quad (8)$$

Definition 11 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)$

Definition 12 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Erealax_2Etrealm_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_lt \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (9)$$

Definition 13 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 14 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (10)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (11)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (12)$$

Definition 15 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{ty_2Enum_2Enum} \quad (13)$$

Definition 16 We define $c_2Emetric_2Eismet$ to be $\lambda A.\lambda 27a : \iota.\lambda V0m \in (ty_2Erealax_2Ereal)^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (14)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (15)$$

Definition 17 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c^{A_27a})$

Let $c_2Erealx_2Etreax_neg : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealx_2Etreax_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal \\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \end{aligned} \quad (16)$$

Definition 18 We define $c_2Erealx_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealx_2Ereal. (ap\ c_2Erealx_2Ereal$

Definition 19 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealx_2Ereal. \lambda V1y \in ty_2Erealx_2Ereal$

Definition 20 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 21 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealx_2Ereal. (ap\ (ap\ (ap\ (c_2Ebool_2ECOND$

Assume the following.

$$True \quad (17)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow \\ True)) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\ A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ p\ V0t)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ nonempty\ A_27c \Rightarrow (\forall V0f \in ((A_27c^{A_27b})^{A_27a}). (\forall V1x \in \\ A_27a. (\forall V2y \in A_27b. ((ap\ (ap\ (c_2Epair_2EUNCURRY\ A_27a \\ A_27b\ A_27c)\ V0f)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V1x)\ V2y))) = \\ (ap\ (ap\ V0f\ V1x)\ V2y)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap (ap c_2Erealax_2Ereal_add V0x) V1y) = (ap (ap c_2Erealax_2Ereal_add \\
& \quad V1y) V0x))))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
& V0x) (ap (ap c_2Erealax_2Ereal_add V1y) V2z)) = (ap (ap c_2Erealax_2Ereal_add \\
& \quad (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) V2z))))))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = V0x))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
& (ap c_2Erealax_2Ereal_neg V0x)) V0x) = (ap c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0)))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (((ap (ap c_2Ereal_2Ereal_sub V0x) V1y) = (ap c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0)) \Leftrightarrow (V0x = V1y))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap c_2Erealax_2Ereal_neg (ap (ap c_2Ereal_2Ereal_sub V0x) \\
& \quad V1y)) = (ap (ap c_2Ereal_2Ereal_sub V1y) V0x))))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (((ap c_2Ereal_2Eabs V0x) = \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \Leftrightarrow (V0x = (ap c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap c_2Ereal_2Eabs (ap c_2Erealax_2Ereal_neg \\
& \quad V0x)) = (ap c_2Ereal_2Eabs V0x)))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
 & (p (ap (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Eabs (ap (ap c_2Erealax_2Ereal_add \\
 & V0x) V1y))) (ap (ap c_2Erealax_2Ereal_add (ap c_2Ereal_2Eabs \\
 & V0x)) (ap c_2Ereal_2Eabs V1y))))))
 \end{aligned} \tag{30}$$

Theorem 1

$$\begin{aligned}
 & (p (ap (c_2Emetric_2Eismet ty_2Erealax_2Ereal) (ap (c_2Epair_2EUNCURRY \\
 & ty_2Erealax_2Ereal ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
 & (\lambda V0x \in ty_2Erealax_2Ereal. (\lambda V1y \in ty_2Erealax_2Ereal. \\
 & (ap c_2Ereal_2Eabs (ap (ap c_2Ereal_2Ereal_sub V1y) V0x))))))
 \end{aligned}$$