

thm_2Emetric_2EMETRIC__TRIANGLE (TMJpZzEiXPLUJLiucziymueDP49E4nrwM2b)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})\ ty_2Erealax_2Ereal) \tag{4}$$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 8 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (ty$

Definition 15 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Definition 16 We define $c_2Emetric_2Eismet$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod A_27a A_27b)}$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Emetric_2Emetric A0) \quad (14)$$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emetric_2Edist A_27a \in ((ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod A_27a A_27b)} (c_2Emetric_2Edist A_27a)) \Rightarrow c_2Emetric_2Edist A_27a) \quad (15)$$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p V0t)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0m \in (ty_2Emetric_2Emetric A_27a).(p (ap (c_2Emetric_2Eismet A_27a) (ap (c_2Emetric_2Edist A_27a) V0m)))) \quad (18)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0m \in (ty_2Emetric_2Emetric A_27a).(\forall V1x \in A_27a.(\forall V2y \in A_27a.((ap (ap (c_2Emetric_2Edist \\ & A_27a) V0m) (ap (ap (c_2Epair_2E_2C A_27a A_27a) V1x) V2y)) = (ap \\ & (ap (c_2Emetric_2Edist A_27a) V0m) (ap (ap (c_2Epair_2E_2C A_27a \\ & A_27a) V2y) V1x)))))) \end{aligned} \quad (19)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0m \in (ty_2Emetric_2Emetric A_27a).(\forall V1x \in A_27a.(\forall V2y \in A_27a.(\forall V3z \in \\ & A_27a.(p (ap (ap c_2Ereal_2Ereal_lte (ap (ap (c_2Emetric_2Edist A_27a) V0m) (ap (ap (c_2Epair_2E_2C A_27a A_27a) V1x) V3z))) (ap \\ & (ap c_2Erealax_2Ereal_add (ap (ap (c_2Emetric_2Edist A_27a) V0m) (ap (ap (c_2Epair_2E_2C A_27a A_27a) V1x) V2y))) (ap (ap (c_2Emetric_2Edist \\ & A_27a) V0m) (ap (ap (c_2Epair_2E_2C A_27a A_27a) V2y) V3z))))))))) \end{aligned}$$