

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Emetric_2Emetric\ A0) \quad (4)$$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Edist\ A_27a \in ((ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}) \quad (5)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (6)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal_REP_CLASS}) \quad (7)$$

Definition 10 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ (ty_2Erealax_2Ereal_REP_CLASS\ a)))$

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (8)$$

Definition 11 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ (ty_2Erealax_2Ereal_lt\ T1\ T2)))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (9)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (10)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (11)$$

Definition 12 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (12)$$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (13)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopology\ A_27a \in ((ty_2Etopology_2Etopology\ A_27a)^{(2^{(2^A_27a)}})) \quad (14)$$

Definition 13 We define $c_Emetric_Eemptop$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_Emetric_Emetric A_27a).(ap$

Definition 14 We define $c_Ebool_E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_Emin_E_3D_3D_3E V0t) c_Ebool_E_7E$

Let $c_Etopology_Eneigh : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Etopology_Eneigh A_27a \in ((2^{(ty_Epair_Eprod (2^{A_27a}) A_27a)})(ty_Etopology_Etopology A_27a)) \quad (15)$$

Definition 15 We define $c_Etopology_Elimpt$ to be $\lambda A_27a : \iota.\lambda V0top \in (ty_Etopology_Etopology A_27a$

Let $c_Erealax_Etrealm_neg : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_neg \in ((ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)) \quad (16)$$

Let $c_Erealax_Etrealm_eq : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_eq \in ((2^{(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)})(ty_Epair_Eprod ty_Ehreal_Ehreal)) \quad (17)$$

Let $c_Erealax_Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_Erealax_Ereal_ABS_CLASS \in (ty_Erealax_Ereal)^{(2^{(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)})} \quad (18)$$

Definition 16 We define $c_Erealax_Ereal_ABS$ to be $\lambda V0r \in (ty_Epair_Eprod ty_Ehreal_Ehreal ty$

Definition 17 We define $c_Erealax_Ereal_neg$ to be $\lambda V0T1 \in ty_Erealax_Ereal.(ap c_Erealax_Ereal$

Let $c_Erealax_Etrealm_add : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_add \in (((ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)))(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal) \quad (19)$$

Definition 18 We define $c_Erealax_Ereal_add$ to be $\lambda V0T1 \in ty_Erealax_Ereal.\lambda V1T2 \in ty_Erealax$

Definition 19 We define $c_Ereal_Ereal_sub$ to be $\lambda V0x \in ty_Erealax_Ereal.\lambda V1y \in ty_Erealax_Ereal$

Definition 20 We define $c_Ereal_Ereal_lte$ to be $\lambda V0x \in ty_Erealax_Ereal.\lambda V1y \in ty_Erealax_Ereal$

Definition 21 We define c_Ebool_ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 22 We define c_Ereal_Eabs to be $\lambda V0x \in ty_Erealax_Ereal.(ap (ap (ap (c_Ebool_ECOND$

Let $c_Epair_EESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_Epair_EESND A_27a A_27b \in (A_27b)^{(ty_Epair_Eprod A_27a A_27b)} \quad (20)$$

Let $c_Epair_EFAST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_Epair_EFAST A_27a A_27b \in (A_27a)^{(ty_Epair_Eprod A_27a A_27b)} \quad (21)$$

Definition 23 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c)^{A_27a})$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Emetric_2Emetric\ A_27a \in ((ty_2Emetric_2Emetric\ A_27a)^{(ty_2Erealax_2Ereal\ (ty_2Epair_2Eprod\ A_27a\ A_27a))}) \quad (22)$$

Definition 24 We define $c_2Emetric_2Emr1$ to be $(ap\ (c_2Emetric_2Emetric\ ty_2Erealax_2Ereal)\ (ap\ (c_2Erealax_2Ereal\ ty_2Erealax_2Ereal)))$

Definition 25 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2ET)$.

Definition 26 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (23)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (24)$$

Definition 27 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num\ m)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (25)$$

Definition 28 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap\ (ap\ c_2Earithmetic_2E_2B\ n))$

Definition 29 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $c_2Erealax_2Etreal_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_inv \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (26)$$

Definition 30 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. (ap\ c_2Erealax_2Ereal_ABS\ T1)$

Let $c_2Erealax_2Etreal_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (27)$$

Definition 31 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. \lambda V1T2 \in ty_2Erealax_2Ereal. (ap\ c_2Erealax_2Ereal_mul\ T1\ T2)$

Definition 32 We define $c_2Ereal_2E_2F$ to be $\lambda V0x \in ty_2Erealax_2Ereal. \lambda V1y \in ty_2Erealax_2Ereal. (ap\ c_2Erealax_2Ereal_mul\ x\ y)$

Assume the following.

$$True \quad (28)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow \forall A.27b.nonempty \ A.27b \Rightarrow (\\ \forall V0t1 \in A.27a. (\forall V1t2 \in A.27b. ((ap (\lambda V2x \in A.27b. \\ V0t1) \ V1t2) = V0t1))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\ (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in \\ A.27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\ (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ p \ V0t)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0t1 \in A.27a. (\forall V1t2 \in \\ A.27a. (((ap (ap (ap (c.2Ebool.2ECOND \ A.27a) \ c.2Ebool.2ET) \ V0t1) \\ V1t2) = V0t1) \wedge ((ap (ap (c.2Ebool.2ECOND \ A.27a) \ c.2Ebool.2EF) \\ V0t1) \ V1t2) = V1t2)))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0m \in (ty.2Emetric.2Emetric \\ A.27a). (\forall V1x \in A.27a. (\forall V2S.27 \in (2^{A.27a}). ((p (ap \\ (ap (ap (c.2Etopology.2Elimpt \ A.27a) (ap (c.2Emetric.2Emtop \ A.27a) \\ V0m)) \ V1x) \ V2S.27)) \Leftrightarrow (\forall V3e \in ty.2Erealax.2Ereal. ((p (ap \\ (ap \ c.2Erealax.2Ereal.lt (ap \ c.2Ereal.2Ereal.of_num \ c.2Enum.2E0) \\ V3e)) \Rightarrow (\exists V4y \in A.27a. ((\neg(V1x = V4y)) \wedge ((p (ap \ V2S.27 \ V4y)) \wedge \\ (p (ap (ap \ c.2Erealax.2Ereal.lt (ap (ap (c.2Emetric.2Edist \ A.27a) \\ V0m) (ap (ap (c.2Epair.2E.2C \ A.27a \ A.27a) \ V1x) \ V4y)))))))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in ty.2Erealax.2Ereal. (\forall V1d \in ty.2Erealax.2Ereal. \\ ((ap (ap (c.2Emetric.2Edist \ ty.2Erealax.2Ereal) \ c.2Emetric.2Emr1) \\ (ap (ap (c.2Epair.2E.2C \ ty.2Erealax.2Ereal \ ty.2Erealax.2Ereal) \\ V0x) (ap (ap \ c.2Erealax.2Ereal.add \ V0x) \ V1d))) = (ap \ c.2Ereal.2Eabs \\ V1d)))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (((ap (ap c_2Erealax_2Ereal_add V0x) V1y) = V0x) \Leftrightarrow (V1y = (ap c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Erealax_2Ereal_lt V0x) V1y)) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\
& \quad V0x) V1y))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Erealax_2Ereal_lt V0x) V1y)) \Rightarrow (\neg(V0x = V1y))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0d \in ty_2Erealax_2Ereal. ((p (ap (ap c_2Erealax_2Ereal_lt \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) (ap (ap c_2Ereal_2E_2F \\
& V0d) (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO)))))) \Leftrightarrow (p (\\
& ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \\
& \quad V0d))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0d \in ty_2Erealax_2Ereal. ((p (ap (ap c_2Erealax_2Ereal_lt \\
& (ap (ap c_2Ereal_2E_2F V0d) (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO)))))) V0d)) \Leftrightarrow \\
& (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0)) V0d))))))
\end{aligned} \tag{40}$$

Theorem 1

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (p (ap (ap (ap (c_2Etopology_2Elimpt \\
& ty_2Erealax_2Ereal) (ap (c_2Emetric_2Emtop ty_2Erealax_2Ereal) \\
& c_2Emetric_2Emr1)) V0x) (c_2Epred_set_2EUNIV ty_2Erealax_2Ereal))))))
\end{aligned}$$