

# thm\_2Enets\_2EDORDER\_\_LEMMA

(TMUURYgvf9eW6PG4fZbwfTLAtmNVRXhLmo2)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2EF$

**Definition 7** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ .

**Definition 8** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A$

**Definition 9** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

**Definition 10** We define  $c\_2Enets\_2Edorder$  to be  $\lambda A\_27a : \iota.\lambda V0g \in ((2^{A-27a})^{A-27a}).(ap (c\_2Ebool\_2E\_21 A$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \tag{2}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \tag{3}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\
& p \ V0t)))))) \quad (4)
\end{aligned}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0g \in ((2^{A\_27a})^{A\_27a}). \\
& ((p \ (ap \ (c\_2Enets\_2Edorder \ A\_27a) \ V0g)) \Rightarrow (\forall V1P \in (2^{A\_27a}). \\
& (\forall V2Q \in (2^{A\_27a}). (((\exists V3n \in A\_27a. ((p \ (ap \ (ap \ V0g \ V3n) \\
V3n)) \wedge (\forall V4m \in A\_27a. ((p \ (ap \ (ap \ V0g \ V4m) \ V3n)) \Rightarrow (p \ (ap \ V1P \ V4m)))))) \wedge \\
& (\exists V5n \in A\_27a. ((p \ (ap \ (ap \ V0g \ V5n) \ V5n)) \wedge (\forall V6m \in A\_27a. \\
& ((p \ (ap \ (ap \ V0g \ V6m) \ V5n)) \Rightarrow (p \ (ap \ V2Q \ V6m)))))) \Rightarrow (\exists V7n \in A\_27a. \\
& ((p \ (ap \ (ap \ V0g \ V7n) \ V7n)) \wedge (\forall V8m \in A\_27a. ((p \ (ap \ (ap \ V0g \ V8m) \\
V7n)) \Rightarrow ((p \ (ap \ V1P \ V8m)) \wedge (p \ (ap \ V2Q \ V8m))))))))))
\end{aligned}$$