

thm_2Enets_2EDORDER__TENDSTO
(TMTEkiEUhY5qmf8mLPtgnFZAnBFQd9tUymu)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})$

Definition 4 We define $c_2Ebool_2E_EF$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_EF$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 8 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t$

Definition 10 We define $c_2Enets_2Edorder$ to be $\lambda A_27a : \iota.\lambda V0g \in ((2^{A_27a})^{A_27a}).(ap (c_2Ebool_2E_21 A$

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty \ ty_2Erealx_2Ereal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty \ A0 \Rightarrow \forall A1.nonempty \ A1 \Rightarrow nonempty \ (ty_2Epair_2Eprod \ A0 \ A1) \tag{2}$$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty \ A0 \Rightarrow nonempty \ (ty_2Emetric_2Emetric \ A0) \tag{3}$$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Edist\ A_27a \in ((ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$$
 (4)

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega$$
 (5)

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum$$
 (6)

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega})$$
 (7)

Definition 11 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum})$$
 (8)

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal$$
 (9)

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal})$$
 (10)

Definition 12 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E40\ ($

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)})$$
 (11)

Definition 13 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}})$$
 (12)

Definition 14 We define c_2Epair_2E2C to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2E$

Let $c_2Enets_2Etendsto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Enets_2Etendsto\ A_27a \in (((2^{A_27a})^{A_27a})^{(ty_2Epair_2Eprod\ (ty_2Emetric_2Edist\ A_27a))})$$
 (13)

Definition 15 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21 2) (\lambda V2t \in$

Definition 16 We define $c_Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Er$

Assume the following.

$$True \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p V0t)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0m \in (ty_2Emetric_2Emetric \\ & A.27a).(\forall V1x \in A.27a.(\forall V2y \in A.27a.(\forall V3z \in \\ & A.27a.((p (ap (ap (ap (c_2Enets_2Etendsto A.27a) (ap (ap (c_2Epair_2E_2C \\ & ty_2Emetric_2Emetric A.27a) A.27a) V0m) V1x)) V2y) V3z)) \Leftrightarrow ((p \\ & (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \\ & (ap (ap (c_2Emetric_2Edist A.27a) V0m) (ap (ap (c_2Epair_2E_2C \\ & A.27a A.27a) V1x) V2y)))) \wedge (p (ap (ap c_2Ereal_2Ereal_lte (ap (\\ & (c_2Emetric_2Edist A.27a) V0m) (ap (ap (c_2Epair_2E_2C A.27a \\ & A.27a) V1x) V2y))) (ap (ap (c_2Emetric_2Edist A.27a) V0m) (ap (ap \\ & (c_2Epair_2E_2C A.27a A.27a) V1x) V3z)))))))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\ & ((p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)) \vee (p (ap (ap c_2Ereal_2Ereal_lte \\ & V1y) V0x)))))) \end{aligned} \quad (18)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(p (ap (ap c_2Ereal_2Ereal_lte V0x) V0x))) \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\ & (\forall V2z \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Ereal_2Ereal_lte \\ & V0x) V1y)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte V1y) V2z))) \Rightarrow (p (ap (\\ & ap c_2Ereal_2Ereal_lte V0x) V2z)))))) \end{aligned} \quad (20)$$

Theorem 1

$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow (\forall V0m \in (ty_2Emetric_2Emetric$
 $A_{27a}).(\forall V1x \in A_{27a}.(p\ (ap\ (c_2Enets_2Edorder\ A_{27a})\ ($
 $ap\ (c_2Enets_2Extendsto\ A_{27a})\ (ap\ (ap\ (c_2Epair_2E_2C\ (ty_2Emetric_2Emetric$
 $A_{27a})\ A_{27a})\ V0m)\ V1x))))))$