thm_2Enets_2ELIM__TENDS (TMHUc1NrdgFHFCWmuqhGgffP6H9UcA8djTz)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 6 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{2}$$

Definition 11 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \tag{3}$$

Let $c\_2Etopology\_2Eneigh : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Eneigh\ A\_27a \in ((2^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ A\_27a)})^{(ty\_2Etopology\_2Etopology\ A\_27a)}) \tag{4}$$

Definition 12 *We define* c_2Etopology_2Elimpt *to be* $\lambda A\_27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Etopology\ A\_2$

Let $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Emetric\_2Emetric\ A0) \tag{5}$$

Let $c\_2Enets\_2Etendsto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Enets\_2Etendsto\ A\_27a \in (((2^{A\_27a})^{A\_27a})^{(ty\_2Epair\_2Eprod\ (ty\_2Emetric\_2Em} \tag{6}$$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{7}$$

Let $c\_2Emetric\_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Edist\ A\_27a \in ((ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27} \tag{8}$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{9}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{10}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{11}$$

Definition 13 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{12}$$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{13}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{14}$$

Definition 14 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 $(t$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{15}$$

Definition 15 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Let $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^{A\_27a})})}) \tag{16}$$

Definition 16 *We define* c_2Emetric_2Emtop *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Emetric\_2Emetric\ A\_27a).(ap$

Let $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Enets\_2Etends\ A\_27a\ A\_27b \in (((2^{(ty\_2Epair\_2Eprod\ (ty\_2Etopology\_2Etopology\ A\_27a)\ ((2^{A\_27b})^{A\_27b}))})^{A\_27a})^{(A\_27a^{A\_27b})}) \tag{17}$$

Definition 17 *We define* c_2Epred__set_2EUNIV *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 18 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Assume the following.

$$True \tag{18}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{19}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{20}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg (p\ V0t)))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27b.((ap\ (\lambda V2x \in A\_27b.\ V0t1)\ V1t2) = V0t1))) \tag{22}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{24}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{25}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0m \in (ty\_2Emetric\_2Emetric\ A\_27a).(\forall V1x \in A\_27a.(\forall V2y \in A\_27a.((ap\ (ap\ (c\_2Emetric\_2Edist\ A\_27a)\ V0m)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ V1x)\ V2y)) = (ap\ (ap\ (c\_2Emetric\_2Edist\ A\_27a)\ V0m)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ V2y)\ V1x)))))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0m \in (ty\_2Emetric\_2Emetric\ A\_27a).(\forall V1x \in A\_27a.(\forall V2y \in A\_27a.((\neg (V1x = V2y)) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ (ap\ (c\_2Emetric\_2Edist\ A\_27a)\ V0m)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ V1x)\ V2y)))))))) \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0m \in (ty\_2Emetric\_2Emetric\ A\_27a).(\forall V1x \in A\_27a.(\forall V2S\_27 \in (2^{A\_27a}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Etopology\_2Elimpt}\ A\_27a)\ (ap\ (\mathsf{c\_2Emetric\_2Emtop}\ A\_27a)\ V0m))\ V1x)\ V2S\_27)) \Leftrightarrow (\forall V3e \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V3e)) \Rightarrow (\exists V4y \in A\_27a.((\neg (V1x = V4y)) \wedge ((p\ (ap\ V2S\_27\ V4y)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ (ap\ (c\_2Emetric\_2Edist\ A\_27a)\ V0m)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ V1x)\ V4y)))\ V3e)))))))))) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0m \in (ty\_2Emetric\_2Emetric\ A\_27a).(\forall V1x \in A\_27a.(\forall V2y \in A\_27a.(\forall V3z \in A\_27a.((p\ (ap\ (ap\ (ap\ (c\_2Enets\_2Etendsto\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Emetric\_2Emetric\ A\_27a)\ A\_27a)\ V0m)\ V1x))\ V2y)\ V3z)) \Leftrightarrow ((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ (ap\ (c\_2Emetric\_2Edist\ A\_27a)\ V0m)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ V1x)\ V2y)))) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ (ap\ (c\_2Emetric\_2Edist\ A\_27a)\ V0m)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ V1x)\ V2y)))\ (ap\ (ap\ (c\_2Emetric\_2Edist\ A\_27a)\ V0m)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ V1x)\ V3z)))))))))) \tag{30}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0d \in (ty\_2Emetric\_2Emetric\ A\_27a).(\forall V1g \in ((2^{A\_27b})^{A\_27b}).(\forall V2x \in (A\_27a^{A\_27b}).(\forall V3x0 \in A\_27a.((p\ (ap\ (ap\ (ap\ (c\_2Enets\_2Etends\ A\_27a\ A\_27b)\ V2x)\ V3x0)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Etopology\_2Etopology\ A\_27a)\ ((2^{A\_27b})^{A\_27b}))\ (ap\ (\mathsf{c\_2Emetric\_2Emtop}\ A\_27a)\ V0d))\ V1g))) \Leftrightarrow (\forall V4e \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V4e)) \Rightarrow (\exists V5n \in A\_27b.((p\ (ap\ (ap\ V1g\ V5n)\ V5n)) \wedge (\forall V6m \in A\_27b.((p\ (ap\ (ap\ V1g\ V6m)\ V5n)) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ (ap\ (c\_2Emetric\_2Edist\ A\_27a)\ V0d)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ (ap\ V2x\ V6m))\ V3x0)))\ V4e))))))))))))) \tag{31}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0x)\ V0x))) \tag{32}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V0x)\ V1y)) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0x)\ V1y))))) \tag{33}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(\forall V2z \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0x)\ V1y)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V1y)\ V2z))) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0x)\ V2z)))))) \tag{34}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0m1 \in (ty\_2Emetric\_2Emetric\ A\_27a).(\forall V1m2 \in (ty\_2Emetric\_2Emetric\ A\_27b).(\forall V2f \in (A\_27b^{A\_27a}).(\forall V3x0 \in A\_27a.(\forall V4y0 \in A\_27b.((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Etopology\_2Elimpt}\ A\_27a)\ (ap\ (\mathsf{c\_2Emetric\_2Emtop}\ A\_27a)\ V0m1))\ V3x0)\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ A\_27a))) \Rightarrow ((p\ (ap\ (ap\ (ap\ (c\_2Enets\_2Etends\ A\_27b\ A\_27a)\ V2f)\ V4y0)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Etopology\_2Etopology\ A\_27b)\ ((2^{A\_27a})^{A\_27a}))\ (ap\ (\mathsf{c\_2Emetric\_2Emtop}\ A\_27b)\ V1m2))\ (ap\ (c\_2Enets\_2Etendsto\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Emetric\_2Emetric\ A\_27a)\ A\_27a)\ V0m1)\ V3x0))))) \Leftrightarrow (\forall V5e \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V5e)) \Rightarrow (\exists V6d \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V6d)) \wedge (\forall V7x \in A\_27a.(((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ (ap\ (c\_2Emetric\_2Edist\ A\_27a)\ V0m1)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ V7x)\ V3x0)))) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ (ap\ (c\_2Emetric\_2Edist\ A\_27a)\ V0m1)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ V7x)\ V3x0)))\ V6d))) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ (ap\ (c\_2Emetric\_2Edist\ A\_27b)\ V1m2)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27b\ A\_27b)\ (ap\ V2f\ V7x))\ V4y0)))\ V5e)))))))))))))))$$