

# thm\_2Enets\_2ELIM\_\_TENDS2 (TMNRLwRjpn- FGbA1aKutbGGoKMkdhoDRkpYt)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Emetric\_2Emetric\ A0) \tag{3}$$

Let  $c\_2Emetric\_2Edist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Edist\ A\_27a \in ((ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}) \tag{4}$$

**Definition 7** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ .

**Definition 8** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A$

Let  $c\_2Enets\_2Etendsto : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Enets\_2Etendsto\ A\_27a \in (((2^{A\_27a})^{A\_27a})^{(ty\_2Epair\_2Eprod\ (ty\_2Emetric\_2Emetric))}) \quad (5)$$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \quad (6)$$

**Definition 9** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.))$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (7)$$

**Definition 10** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.))$

Let  $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Enets\_2Etends\ A\_27a\ A\_27b \in (((2^{(ty\_2Epair\_2Eprod\ (ty\_2Etopology\_2Etopology\ A\_27a)\ ((2^{A\_27b})^{A\_27b}))})^{A\_27a})^{(A\_27a)^{A\_27b}}) \quad (8)$$

**Definition 11** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2E\_21\ 2)$ .

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (9)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal\_REP\_CLASS}) \quad (10)$$

**Definition 12** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal\_REP.(ap\ (c\_2Emin\_2E\_40\ 2)\ (\lambda V1t \in 2.))$

Let  $c\_2Erealax\_2Etrealt\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealt\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (11)$$

**Definition 13** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal\_lt.\lambda V1T2 \in ty\_2Erealax\_2Ereal\_lt.$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (12)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (13)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (14)$$

**Definition 14** We define  $c\_Enum\_2E0$  to be (ap  $c\_Enum\_2EABS\_num$   $c\_Enum\_2EZERO\_REP$ ).

Let  $c\_Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (15)$$

Let  $c\_Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow c\_Etopology\_2Etopology \ A\_27a \in ((ty\_2Etopology\_2Etopology \ A\_27a)^{(2^{(2^A-27a)})}) \quad (16)$$

**Definition 15** We define  $c\_Emetric\_2Emtop$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Emetric\_2Emetric \ A\_27a).$ (ap

Let  $c\_Etopology\_2Eneigh : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow c\_Etopology\_2Eneigh \ A\_27a \in ((2^{(ty\_2Epair\_2Eprod \ (2^A-27a) \ A\_27a)} \ (ty\_2Etopology\_2Etopology \ A\_27a))) \quad (17)$$

**Definition 16** We define  $c\_Etopology\_2Elimpt$  to be  $\lambda A\_27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Etopology \ A\_27a)$

**Definition 17** We define  $c\_Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 18** We define  $c\_Earithmetic\_2EZERO$  to be  $c\_Enum\_2E0$ .

Let  $c\_Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (18)$$

Let  $c\_Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (19)$$

**Definition 19** We define  $c\_Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.$ (ap  $c\_Enum\_2EABS\_num$

Let  $c\_Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (20)$$

**Definition 20** We define  $c\_Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.$ (ap (ap  $c\_Earithmetic$

**Definition 21** We define  $c\_Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_Erealax\_2Etrealm\_inv : \iota$  be given. Assume the following.

$$c\_Erealax\_2Etrealm\_inv \in ((ty\_2Epair\_2Eprod \ ty\_2Ehreal\_2Ehreal \ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod \ ty\_2Ehreal\_2Ehreal \ ty\_2Ehreal\_2Ehreal)}) \quad (21)$$

Let  $c\_Erealax\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_Erealax\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod \ ty\_2Ehreal\_2Ehreal \ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod \ ty\_2Ehreal\_2Ehreal \ ty\_2Ehreal\_2Ehreal)}) \quad (22)$$

Let  $c\_Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod \ ty\_2Ehreal\_2Ehreal \ ty\_2Ehreal\_2Ehreal)})}) \quad (23)$$

**Definition 22** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 23** We define  $c\_2Erealax\_2Einv$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal\_ABS$

Let  $c\_2Erealax\_2Etrealmul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealmul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)) (24)$$

**Definition 24** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

**Definition 25** We define  $c\_2Ereal\_2E\_2F$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$

Assume the following.

$$True \quad (25)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (28)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \forall V0m1 \in (ty\_2Emetric\_2Emetric\ A\_27a).(\forall V1m2 \in (ty\_2Emetric\_2Emetric \\
& \quad A\_27b).(\forall V2f \in (A\_27b^{A\_27a}).(\forall V3x0 \in A\_27a.(\forall V4y0 \in \\
& \quad A\_27b.((p\ (ap\ (ap\ (ap\ (c\_2Etopology\_2Elimpt\ A\_27a)\ (ap\ (c\_2Emetric\_2Emtop \\
& \quad \quad A\_27a)\ V0m1))\ V3x0)\ (c\_2Epred\_set\_2EUNIV\ A\_27a)))) \Rightarrow ((p\ (ap\ (ap \\
& \quad \quad (ap\ (c\_2Enets\_2Etends\ A\_27b\ A\_27a)\ V2f)\ V4y0)\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\
& \quad \quad (ty\_2Etopology\_2Etopology\ A\_27b)\ ((2^{A\_27a})^{A\_27a}))\ (ap\ (c\_2Emetric\_2Emtop \\
& \quad \quad A\_27b)\ V1m2))\ (ap\ (c\_2Enets\_2Etendsto\ A\_27a)\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\
& \quad \quad \quad (ty\_2Emetric\_2Emetric\ A\_27a)\ A\_27a)\ V0m1)\ V3x0)))))) \Leftrightarrow (\forall V5e \in \\
& \quad ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& \quad \quad c\_2Enum\_2E0))\ V5e)) \Rightarrow (\exists V6d \in ty\_2Erealax\_2Ereal.((p\ (ap \\
& \quad \quad (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0)) \\
& \quad \quad \quad V6d)) \wedge (\forall V7x \in A\_27a.(((p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt \\
& \quad \quad (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))\ (ap\ (ap\ (c\_2Emetric\_2Edist \\
& \quad \quad \quad A\_27a)\ V0m1)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27a)\ V7x)\ V3x0)))) \wedge \\
& \quad \quad \quad (p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte\ (ap\ (ap\ (c\_2Emetric\_2Edist\ A\_27a) \\
& \quad \quad \quad V0m1)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27a)\ V7x)\ V3x0))))\ V6d))) \Rightarrow \\
& \quad \quad \quad (p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ (ap\ (c\_2Emetric\_2Edist\ A\_27b) \\
& \quad \quad \quad V1m2)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27b\ A\_27b)\ (ap\ V2f\ V7x))\ V4y0))) \\
& \quad \quad \quad V5e))))))))))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ V0x)\ V1y)) \Rightarrow (p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte \\
& \quad V0x)\ V1y))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte \\
& \quad V0x)\ V1y)) \wedge (p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ V1y)\ V2z))) \Rightarrow (p\ (ap \\
& \quad (ap\ c\_2Erealax\_2Ereal\_lt\ V0x)\ V2z))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& (\forall V0d \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt \\
& \quad (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))\ (ap\ (ap\ c\_2Ereal\_2E\_2F \\
& \quad V0d)\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ (ap\ c\_2Earithmetic\_2ENUMERAL \\
& \quad \quad (ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO)))))) \Leftrightarrow (p\ ( \\
& \quad ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0)) \\
& \quad \quad V0d))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& (\forall V0d \in ty\_2Erealax\_2Ereal.((p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& (ap (ap c\_2Ereal\_2E\_2F V0d) (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmic\_2ENUMERAL \\
& (ap c\_2Earithmic\_2EBIT2 c\_2Earithmic\_2EZERO)))))) V0d)) \Leftrightarrow \\
& (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) V0d)))
\end{aligned} \tag{33}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( \\
& \forall V0m1 \in (ty\_2Emetric\_2Emetric A\_27a).(\forall V1m2 \in (ty\_2Emetric\_2Emetric \\
& A\_27b).(\forall V2f \in (A\_27b^{A\_27a}).(\forall V3x0 \in A\_27a.(\forall V4y0 \in \\
& A\_27b.((p (ap (ap (ap (c\_2Etopology\_2Elimpt A\_27a) (ap (c\_2Emetric\_2Emtop \\
& A\_27a) V0m1)) V3x0) (c\_2Epred\_set\_2EUNIV A\_27a)))) \Rightarrow ((p (ap (ap \\
& (ap (c\_2Enets\_2Etends A\_27b A\_27a) V2f) V4y0) (ap (ap (c\_2Epair\_2E\_2C \\
& (ty\_2Etopology\_2Etopology A\_27b) ((2^{A\_27a})^{A\_27a})) (ap (c\_2Emetric\_2Emtop \\
& A\_27b) V1m2)) (ap (c\_2Enets\_2Etendsto A\_27a) (ap (ap (c\_2Epair\_2E\_2C \\
& (ty\_2Emetric\_2Emetric A\_27a) A\_27a) V0m1) V3x0)))))) \Leftrightarrow (\forall V5e \in \\
& ty\_2Erealax\_2Ereal.((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) V5e)) \Rightarrow (\exists V6d \in ty\_2Erealax\_2Ereal.((p (ap \\
& (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) \\
& V6d)) \wedge (\forall V7x \in A\_27a.(((p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) (ap (ap (c\_2Emetric\_2Edist \\
& A\_27a) V0m1) (ap (ap (c\_2Epair\_2E\_2C A\_27a A\_27a) V7x) V3x0)))) \wedge \\
& (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap (ap (c\_2Emetric\_2Edist A\_27a) \\
& V0m1) (ap (ap (c\_2Epair\_2E\_2C A\_27a A\_27a) V7x) V3x0))) V6d))) \Rightarrow \\
& (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap (ap (c\_2Emetric\_2Edist A\_27b) \\
& V1m2) (ap (ap (c\_2Epair\_2E\_2C A\_27b A\_27b) (ap V2f V7x)) V4y0))) \\
& V5e)))))))))))))
\end{aligned}$$