

thm_2Enets_2EMR1__BOUNDED
(TMK7KHTm5QNLc2CLRdCmKq4PSv5xEwiEuLK)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})\ ty_2Erealax_2Ereal) \tag{4}$$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 8 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (ty$

Let $c_2Erealax_2Etreal_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)) \quad (5)$$

Let $c_2Erealax_2Etreal_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \quad (6)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}} \quad (7)$$

Definition 9 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 10 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal)$

Let $c_2Erealax_2Etreal_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal))^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (8)$$

Definition 11 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 12 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (9)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (10)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (11)$$

Definition 13 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{ty_2Enum_2Enum} \quad (12)$$

Let $c_2Erealax_2Etreal_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \quad (13)$$

Let $c_2Enets_2Ebunded : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Enets_2Ebunded \\ & A_27a\ A_27b \in ((2^{(A_27a^{A_27b})})(ty_2Epair_2Eprod\ (ty_2Emetric_2Emetric\ A_27a)\ ((2^{A_27b})^{A_27b}))) \end{aligned} \quad (20)$$

Assume the following.

$$True \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ & V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ & A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\ & ((ap\ (ap\ (c_2Emetric_2Edist\ ty_2Erealax_2Ereal)\ c_2Emetric_2Emr1) \\ & (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal) \\ & V0x)\ V1y)) = (ap\ c_2Ereal_2Eabs\ (ap\ (ap\ c_2Ereal_2Ereal_sub\ V1y) \\ & V0x)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0m \in (ty_2Emetric_2Emetric\ A_27a).(\forall V1g \in ((2^{A_27b})^{A_27b}). \\ & (\forall V2f \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (c_2Enets_2Ebunded\ A_27a \\ & A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ (ty_2Emetric_2Emetric\ A_27a)\ (\\ & (2^{A_27b})^{A_27b}))\ V0m)\ V1g))\ V2f)) \Leftrightarrow (\exists V3k \in ty_2Erealax_2Ereal. \\ & (\exists V4x \in A_27a.(\exists V5N \in A_27b.((p\ (ap\ (ap\ V1g\ V5N)\ V5N)) \wedge \\ & (\forall V6n \in A_27b.((p\ (ap\ (ap\ V1g\ V6n)\ V5N)) \Rightarrow (p\ (ap\ (ap\ c_2Erealax_2Ereal_lt \\ & (ap\ (ap\ (c_2Emetric_2Edist\ A_27a)\ V0m)\ (ap\ (ap\ (c_2Epair_2E_2C \\ & A_27a\ A_27a)\ (ap\ V2f\ V6n))\ V4x)))))))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap (ap c_2Erealax_2Ereal_add V0x) V1y) = (ap (ap c_2Erealax_2Ereal_add \\
& V1y) V0x))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((p (ap (ap c_2Erealax_2Ereal_lt \\
& (ap (ap c_2Erealax_2Ereal_add V0x) V2z)) (ap (ap c_2Erealax_2Ereal_add \\
& V1y) V2z))) \Leftrightarrow (p (ap (ap c_2Erealax_2Ereal_lt V0x) V1y))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. (((p (ap (ap c_2Ereal_2Ereal_lte \\
& V0x) V1y)) \wedge (p (ap (ap c_2Erealax_2Ereal_lt V1y) V2z))) \Rightarrow (p (ap \\
& (ap c_2Erealax_2Ereal_lt V0x) V2z))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap (ap c_2Erealax_2Ereal_add (ap (ap c_2Ereal_2Ereal_sub \\
& V0x) V1y)) V1y) = V0x)))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Ereal_2Ereal_sub \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0) V0x) = (ap c_2Erealax_2Ereal_neg \\
& V0x)))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap c_2Ereal_2Eabs (ap c_2Erealax_2Ereal_neg \\
& V0x)) = (ap c_2Ereal_2Eabs V0x)))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Eabs (ap (ap c_2Erealax_2Ereal_add \\
& V0x) V1y))) (ap (ap c_2Erealax_2Ereal_add (ap c_2Ereal_2Eabs \\
& V0x) (ap c_2Ereal_2Eabs V1y))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap c_2Ereal_2Eabs (ap (ap c_2Ereal_2Ereal_sub V0x) V1y)) = (\\
& ap c_2Ereal_2Eabs (ap (ap c_2Ereal_2Ereal_sub V1y) V0x))))))
\end{aligned} \tag{35}$$

Theorem 1

$$\begin{aligned} & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0g \in ((2^{A_{27a}})^{A_{27a}}). \\ & (\forall V1f \in (\text{ty_2Erealax_2Ereal}^{A_{27a}}). ((p (ap (ap (c_2Enets_2Ebouned \\ & \text{ty_2Erealax_2Ereal } A_{27a}) (ap (ap (c_2Epair_2E_2C (\text{ty_2Emetric_2Emetric} \\ & \text{ty_2Erealax_2Ereal}) ((2^{A_{27a}})^{A_{27a}})) c_2Emetric_2Emr1) V0g)) \\ & V1f)) \Leftrightarrow (\exists V2k \in \text{ty_2Erealax_2Ereal}. (\exists V3N \in A_{27a}. \\ & ((p (ap (ap V0g V3N) V3N)) \wedge (\forall V4n \in A_{27a}. ((p (ap (ap V0g V4n) \\ & V3N)) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Eabs (ap \\ & V1f V4n))) V2k)))))))))) \end{aligned}$$