

thm_2Enets_2ENET__ADD
(TMGZwmq3ykKrHEJZmsDMGkSLzo5Vgxtcend)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_21$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \tag{4}$$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge p)$ of type $\iota \Rightarrow \iota$).

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})))$

Definition 5 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (ty$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal}) \tag{5}$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \quad (6)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}} \quad (7)$$

Definition 6 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 7 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_ABS\ T1)$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal))^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (8)$$

Definition 8 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 9 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (9)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (10)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (11)$$

Definition 10 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{ty_2Enum_2Enum} \quad (12)$$

Let $c_2Erealax_2Etrealm_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \quad (13)$$

Definition 11 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 12 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 13 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0g \in ((2^{A.27a})^{A.27a}). \\
& ((p\ (ap\ (c.2Enets.2Edorder\ A.27a)\ V0g)) \Rightarrow (\forall V1x \in (ty.2Erealax.2Ereal^{A.27a}). \\
& (\forall V2y \in (ty.2Erealax.2Ereal^{A.27a}). ((p\ (ap\ (ap\ (ap\ (c.2Enets.2Etends \\
& ty.2Erealax.2Ereal\ A.27a)\ V1x)\ (ap\ c.2Ereal.2Ereal_of_num \\
& c.2Enum.2E0))\ (ap\ (ap\ (c.2Epair.2E.2C\ (ty.2Etopology.2Etopology \\
& ty.2Erealax.2Ereal)\ ((2^{A.27a})^{A.27a}))\ (ap\ (c.2Emetric.2Emtop \\
& ty.2Erealax.2Ereal)\ c.2Emetric.2Emr1))\ V0g))) \wedge (p\ (ap\ (ap\ (ap \\
& (c.2Enets.2Etends\ ty.2Erealax.2Ereal\ A.27a)\ V2y)\ (ap\ c.2Ereal.2Ereal_of_num \\
& c.2Enum.2E0))\ (ap\ (ap\ (c.2Epair.2E.2C\ (ty.2Etopology.2Etopology \\
& ty.2Erealax.2Ereal)\ ((2^{A.27a})^{A.27a}))\ (ap\ (c.2Emetric.2Emtop \\
& ty.2Erealax.2Ereal)\ c.2Emetric.2Emr1))\ V0g)))) \Rightarrow (p\ (ap\ (ap\ (ap \\
& (c.2Enets.2Etends\ ty.2Erealax.2Ereal\ A.27a)\ (\lambda V3n \in A.27a. \\
& (ap\ (ap\ c.2Erealax.2Ereal_add\ (ap\ V1x\ V3n))\ (ap\ V2y\ V3n))))\ (ap \\
& c.2Ereal.2Ereal_of_num\ c.2Enum.2E0))\ (ap\ (ap\ (c.2Epair.2E.2C \\
& (ty.2Etopology.2Etopology\ ty.2Erealax.2Ereal)\ ((2^{A.27a})^{A.27a})) \\
& (ap\ (c.2Emetric.2Emtop\ ty.2Erealax.2Ereal)\ c.2Emetric.2Emr1)) \\
& V0g)))))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty.2Erealax.2Ereal. (\forall V1y \in ty.2Erealax.2Ereal. \\
& ((ap\ (ap\ c.2Erealax.2Ereal_add\ V0x)\ V1y) = (ap\ (ap\ c.2Erealax.2Ereal_add \\
& V1y)\ V0x))))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty.2Erealax.2Ereal. (\forall V1y \in ty.2Erealax.2Ereal. \\
& (\forall V2z \in ty.2Erealax.2Ereal. ((ap\ (ap\ c.2Erealax.2Ereal_add \\
& V0x)\ (ap\ (ap\ c.2Erealax.2Ereal_add\ V1y)\ V2z)) = (ap\ (ap\ c.2Erealax.2Ereal_add \\
& (ap\ (ap\ c.2Erealax.2Ereal_add\ V0x)\ V1y))\ V2z))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty.2Erealax.2Ereal. (\forall V1y \in ty.2Erealax.2Ereal. \\
& ((ap\ c.2Erealax.2Ereal_neg\ (ap\ (ap\ c.2Erealax.2Ereal_add\ V0x) \\
& V1y)) = (ap\ (ap\ c.2Erealax.2Ereal_add\ (ap\ c.2Erealax.2Ereal_neg \\
& V0x))\ (ap\ c.2Erealax.2Ereal_neg\ V1y))))))
\end{aligned} \tag{29}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{30}$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{31}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False))) \quad (34)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r))) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (35)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (36)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (37)$$

Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0g \in ((2^{A.27a})^{A.27a}). \\ & ((p (ap (c.2Enets.2Eorder \ A.27a) \ V0g)) \Rightarrow (\forall V1x \in (ty.2Erealax.2Ereal^{A.27a}). \\ & (\forall V2x0 \in ty.2Erealax.2Ereal. (\forall V3y \in (ty.2Erealax.2Ereal^{A.27a}). \\ & (\forall V4y0 \in ty.2Erealax.2Ereal. (((p (ap (ap (ap (c.2Enets.2Etends \\ & ty.2Erealax.2Ereal \ A.27a) \ V1x) \ V2x0) (ap (ap (c.2Epair.2E.2C (\\ & ty.2Etopology.2Etopology \ ty.2Erealax.2Ereal) ((2^{A.27a})^{A.27a})) \\ & (ap (c.2Emetric.2Emtop \ ty.2Erealax.2Ereal) \ c.2Emetric.2Emr1)) \\ & V0g))) \wedge (p (ap (ap (ap (c.2Enets.2Etends \ ty.2Erealax.2Ereal \ A.27a) \\ & V3y) \ V4y0) (ap (ap (c.2Epair.2E.2C (ty.2Etopology.2Etopology \\ & ty.2Erealax.2Ereal) ((2^{A.27a})^{A.27a})) (ap (c.2Emetric.2Emtop \\ & ty.2Erealax.2Ereal) \ c.2Emetric.2Emr1)) \ V0g)))) \Rightarrow (p (ap (ap (ap \\ & (c.2Enets.2Etends \ ty.2Erealax.2Ereal \ A.27a) (\lambda V5n \in A.27a. \\ & (ap (ap \ c.2Erealax.2Ereal.add (ap \ V1x \ V5n)) (ap \ V3y \ V5n)))) (ap \\ & (ap \ c.2Erealax.2Ereal.add \ V2x0) \ V4y0)) (ap (ap (c.2Epair.2E.2C \\ & (ty.2Etopology.2Etopology \ ty.2Erealax.2Ereal) ((2^{A.27a})^{A.27a})) \\ & (ap (c.2Emetric.2Emtop \ ty.2Erealax.2Ereal) \ c.2Emetric.2Emr1)) \\ & V0g)))))) \end{aligned}$$