

thm_2Enets_2ENET__CONV__BOUNDED
(TMHGwD2aUAPsnFpJUw8GiFK9aH8zyrRvQDk)

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Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})\ ty_2Erealax_2Ereal) \tag{4}$$

Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p\ (ap\ P\ x))$ **then** (the $(\lambda x.x \in A \wedge p)$ of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_21$ to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a})))$

Definition 5 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ (ty_2Erealax_2Ereal_REP_CLASS\ a)))$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)\ (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)) \tag{5}$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (6)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (7)$$

Definition 6 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 7 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_ABS)$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (8)$$

Definition 8 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 9 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (9)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (10)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (11)$$

Definition 10 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{ty_2Enum_2Enum} \quad (12)$$

Let $c_2Erealax_2Etrealm_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (13)$$

Definition 11 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 12 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 13 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 14 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 15 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Definition 16 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2.))$

Definition 17 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.))$

Definition 18 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap (ap (ap (c_2Ebool_2ECOND$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (14)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (15)$$

Definition 19 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a}$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Emetric_2Emetric\ A0) \quad (16)$$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Emetric\ A_27a \in ((ty_2Emetric_2Emetric \\ A_27a)^{(ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})}) \end{aligned} \quad (17)$$

Definition 20 We define $c_2Emetric_2Emr1$ to be $(ap (c_2Emetric_2Emetric\ ty_2Erealax_2Ereal) (ap (c$

Definition 21 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (18)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (19)$$

Definition 22 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (20)$$

Definition 23 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic$

Definition 24 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Enets_2Ebunded : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Enets_2Ebunded \\ & A_27a A_27b \in ((2^{(A_27a^{A_27b})})(ty_2Epair_2Eprod (ty_2Emetric_2Emetric A_27a) ((2^{A_27b})^{A_27b}))) \end{aligned} \quad (21)$$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emetric_2Edist A_27a \in ((ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod A_27a A_27a)})) \quad (22)$$

Definition 25 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod \\ & A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{(A_27b)^{A_27a}})}) \end{aligned} \quad (23)$$

Definition 26 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b.(ap (c_2E$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Etopology_2Etopology A0) \quad (24)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow c_2Etopology_2Etopology A_27a \in \\ & ((ty_2Etopology_2Etopology A_27a)^{(2^{(2^{A_27a})})}) \end{aligned} \quad (25)$$

Definition 27 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota. \lambda V0m \in (ty_2Emetric_2Emetric A_27a).(ap$

Let $c_2Enets_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Enets_2Etends \\ & A_27a A_27b \in (((2^{(ty_2Epair_2Eprod (ty_2Etopology_2Etopology A_27a) ((2^{A_27b})^{A_27b})))})_{A_27a})_{(A_27a^{A_27b})} \end{aligned} \quad (26)$$

Definition 28 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Assume the following.

$$\begin{aligned} & ((ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)) = \\ & (ap c_2Enum_2ESUC c_2Enum_2E0)) \end{aligned} \quad (27)$$

Assume the following.

$$True \quad (28)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (32)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p\ V0t)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0m \in (ty_2Emetric_2Emetric\ A_27a). (\forall V1g \in ((2^{A_27b})^{A_27b}). \\ & (\forall V2f \in (A_27a^{A_27b}). ((p\ (ap\ (ap\ (c_2Enets_2Ebounded\ A_27a \\ & A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ (ty_2Emetric_2Emetric\ A_27a)\ (\\ & (2^{A_27b})^{A_27b}))\ V0m)\ V1g))\ V2f)) \Leftrightarrow (\exists V3k \in ty_2Erealx_2Ereal. \\ & (\exists V4x \in A_27a. (\exists V5N \in A_27b. ((p\ (ap\ (ap\ V1g\ V5N)\ V5N)) \wedge \\ & (\forall V6n \in A_27b. ((p\ (ap\ (ap\ V1g\ V6n)\ V5N)) \Rightarrow (p\ (ap\ (ap\ c_2Erealx_2Ereal_lt \\ & (ap\ (ap\ (c_2Emetric_2Edist\ A_27a)\ V0m)\ (ap\ (ap\ (c_2Epair_2E_2C \\ & A_27a\ A_27a)\ (ap\ V2f\ V6n))\ V4x))) V3k)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0d \in (ty_2Emetric_2Emetric\ A_27a).(\forall V1g \in ((2^{A_27b})^{A_27b}). \\
& \quad (\forall V2x \in (A_27a^{A_27b}).(\forall V3x0 \in A_27a.((p\ (ap\ (ap\ (ap \\
& \quad (c_2Enets_2Etends\ A_27a\ A_27b)\ V2x)\ V3x0)\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad (ty_2Etopology_2Etopology\ A_27a)\ ((2^{A_27b})^{A_27b}))\ (ap\ (c_2Emetric_2Emtop \\
& \quad A_27a)\ V0d))\ V1g))) \Leftrightarrow (\forall V4e \in ty_2Erealax_2Ereal.((p\ (ap \\
& \quad (ap\ c_2Erealax_2Ereal_lt\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) \\
& \quad V4e)) \Rightarrow (\exists V5n \in A_27b.((p\ (ap\ (ap\ V1g\ V5n)\ V5n)) \wedge (\forall V6m \in \\
& \quad A_27b.((p\ (ap\ (ap\ V1g\ V6m)\ V5n)) \Rightarrow (p\ (ap\ (ap\ c_2Erealax_2Ereal_lt \\
& \quad (ap\ (ap\ (c_2Emetric_2Edist\ A_27a)\ V0d)\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad A_27a\ A_27a)\ (ap\ V2x\ V6m))\ V3x0)))\ V4e))))))))) \\
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum.(p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Enum_2E0) \\
& \quad (ap\ c_2Enum_2ESUC\ V0n)))) \\
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\
& \quad (p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ (ap\ c_2Ereal_2Ereal_of_num \\
& \quad V0m))\ (ap\ c_2Ereal_2Ereal_of_num\ V1n)))) \Leftrightarrow (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& \quad V0m)\ V1n)))) \\
\end{aligned} \tag{37}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0g \in ((2^{A_27a})^{A_27a}). \\
& \quad (\forall V1x \in (ty_2Erealax_2Ereal^{A_27a}).(\forall V2x0 \in ty_2Erealax_2Ereal. \\
& \quad ((p\ (ap\ (ap\ (ap\ (c_2Enets_2Etends\ ty_2Erealax_2Ereal\ A_27a)\ V1x) \\
& \quad V2x0)\ (ap\ (ap\ (c_2Epair_2E_2C\ (ty_2Etopology_2Etopology\ ty_2Erealax_2Ereal) \\
& \quad ((2^{A_27a})^{A_27a}))\ (ap\ (c_2Emetric_2Emtop\ ty_2Erealax_2Ereal) \\
& \quad c_2Emetric_2Emr1))\ V0g))) \Rightarrow (p\ (ap\ (ap\ (c_2Enets_2Ebounded\ ty_2Erealax_2Ereal \\
& \quad A_27a)\ (ap\ (ap\ (c_2Epair_2E_2C\ (ty_2Emetric_2Emetric\ ty_2Erealax_2Ereal) \\
& \quad ((2^{A_27a})^{A_27a}))\ c_2Emetric_2Emr1)\ V0g))\ V1x)))))) \\
\end{aligned}$$