

thm_2Enets_2ENET_INV (TMVeFBGSGC- TamnSV1Khm1FvUkDLJAYTX7Be)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 6 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 9 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a P))$

Definition 10 We define $c_2Enets_2Edorder$ to be $\lambda A_27a : \iota.\lambda V0g \in ((2^{A_27a})^{A_27a}).(ap (c_2Ebool_2E_21 A_27a g))$

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty \ ty_2Erealx_2Ereal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty \ A0 \Rightarrow \forall A1.nonempty \ A1 \Rightarrow nonempty \ (ty_2Epair_2Eprod \ A0 \ A1) \tag{2}$$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty \ A0 \Rightarrow nonempty \ (ty_2Emetric_2Emetric \ A0) \tag{3}$$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Edist\ A_27a \in ((ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) (4))$$

Let $c_2Enets_2Ebunded : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Enets_2Ebunded\ A_27a\ A_27b \in ((2^{(A_27a\ A_27b)}) (ty_2Epair_2Eprod\ (ty_2Emetric_2Emetric\ A_27a)\ ((2^{A_27b})^{A_27b}))) (5)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal (6)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) ty_2Erealax_2Ereal (7))$$

Definition 11 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E40\ ty_2Erealax_2Ereal\ V0a))$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) (8)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)) (9)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})}) (10)$$

Definition 12 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 13 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_neg\ V0T1)$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)) (11)$$

Definition 14 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 15 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{12}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{13}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{14}$$

Definition 16 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \tag{15}$$

Let $c_2Erealax_2Ereal_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \tag{16}$$

Definition 17 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 18 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E7E))$

Definition 19 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Definition 20 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.))$

Definition 21 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ (ap\ (c_2Ebool_2ECOND$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \tag{17}$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \tag{18}$$

Definition 22 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a})$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Emetric\ A_27a \in ((ty_2Emetric_2Emetric \\ A_27a)^{(ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})}) \end{aligned} \tag{19}$$

Definition 23 We define $c_2Emetric_2Emr1$ to be $(ap (c_2Emetric_2Emetric ty_2Erealx_2Ereal) (ap (c_2$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (20)$$

Definition 24 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Etopology_2Etopology A0) \quad (21)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Etopology_2Etopology A_27a \in ((ty_2Etopology_2Etopology A_27a)^{(2^{(2^{A_27a})})}) \quad (22)$$

Definition 25 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota. \lambda V0m \in (ty_2Emetric_2Emetric A_27a). (ap$

Let $c_2Enets_2Eextends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Enets_2Eextends A_27a A_27b \in (((2^{(ty_2Epair_2Eprod (ty_2Etopology_2Etopology A_27a) ((2^{A_27b})^{A_27b}))})_{A_27a})^{(A_27a)^{A_27b}}) \quad (23)$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (24)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (25)$$

Definition 26 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap c_2Enum_2EABS_num$

Definition 27 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 28 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})_{ty_2Enum_2Enum} \quad (26)$$

Definition 29 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic$

Definition 30 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $c_2Erealax_2Etrealm_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_inv \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (27)$$

Definition 31 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_ABS$

Definition 32 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2$

Let $c_2Erealax_2Etrealm_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (28)$$

Definition 33 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Assume the following.

$$((ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)) = (ap\ c_2Enum_2ESUC\ c_2Enum_2E0)) \quad (29)$$

Assume the following.

$$True \quad (30)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (31)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (32)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (33)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (34)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (35)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (36)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (37)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (38)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (39)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0t1 \in A.27a.(\forall V1t2 \in A.27a.(((ap (ap (ap (c.2Ebool.2ECOND A.27a) c.2Ebool.2ET) V0t1) V1t2) = V0t1) \wedge ((ap (ap (ap (c.2Ebool.2ECOND A.27a) c.2Ebool.2EF) V0t1) V1t2) = V1t2)))) \quad (40)$$

Assume the following.

$$(\forall V0x \in ty.2Erealax.2Ereal.(\forall V1y \in ty.2Erealax.2Ereal.(((ap (ap (c.2Emetric.2Edist ty.2Erealax.2Ereal) c.2Emetric.2Emr1) (ap (ap (c.2Epair.2E_2C ty.2Erealax.2Ereal ty.2Erealax.2Ereal) V0x) V1y)) = (ap c.2Ereal.2Eabs (ap (ap c.2Ereal.2Ereal._sub V1y) V0x)))))) \quad (41)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0g \in ((2^{A.27a})^{A.27a}).(((p (ap (c.2Enets.2Edorder A.27a) V0g)) \Rightarrow (\forall V1P \in (2^{A.27a}).(\forall V2Q \in (2^{A.27a}).(((\exists V3n \in A.27a.((p (ap (ap V0g V3n) V3n)) \wedge (\forall V4m \in A.27a.((p (ap (ap V0g V4m) V3n)) \Rightarrow (p (ap V1P V4m)))))) \wedge (\exists V5n \in A.27a.((p (ap (ap V0g V5n) V5n)) \wedge (\forall V6m \in A.27a.((p (ap (ap V0g V6m) V5n)) \Rightarrow (p (ap V2Q V6m)))))) \Rightarrow (\exists V7n \in A.27a.((p (ap (ap V0g V7n) V7n)) \wedge (\forall V8m \in A.27a.((p (ap (ap V0g V8m) V7n)) \Rightarrow ((p (ap V1P V8m)) \wedge (p (ap V2Q V8m)))))))))))))) \quad (42)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0d \in (ty_2Emetric_2Emetric\ A_27a).(\forall V1g \in ((2^{A_27b})^{A_27b}). \\
& \quad (\forall V2x \in (A_27a^{A_27b}).(\forall V3x0 \in A_27a.((p\ (ap\ (ap\ (ap \\
& \quad (c_2Enets_2Etends\ A_27a\ A_27b)\ V2x)\ V3x0)\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad (ty_2Etopology_2Etopology\ A_27a)\ ((2^{A_27b})^{A_27b}))\ (ap\ (c_2Emetric_2Emtop \\
& \quad A_27a)\ V0d))\ V1g))) \Leftrightarrow (\forall V4e \in ty_2Erealax_2Ereal.((p\ (ap \\
& \quad (ap\ c_2Erealax_2Ereal_lt\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) \\
& \quad V4e)) \Rightarrow (\exists V5n \in A_27b.((p\ (ap\ (ap\ V1g\ V5n)\ V5n)) \wedge (\forall V6m \in \\
& \quad A_27b.((p\ (ap\ (ap\ V1g\ V6m)\ V5n)) \Rightarrow (p\ (ap\ (ap\ c_2Erealax_2Ereal_lt \\
& \quad (ap\ (ap\ (c_2Emetric_2Edist\ A_27a)\ V0d)\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad A_27a\ A_27a)\ (ap\ V2x\ V6m))\ V3x0)))\ V4e))))))))) \\
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0g \in ((2^{A_27a})^{A_27a}). \\
& \quad (\forall V1f \in (ty_2Erealax_2Ereal^{A_27a}).((p\ (ap\ (ap\ (c_2Enets_2Ebouned \\
& \quad ty_2Erealax_2Ereal\ A_27a)\ (ap\ (ap\ (c_2Epair_2E_2C\ (ty_2Emetric_2Emetric \\
& \quad ty_2Erealax_2Ereal)\ ((2^{A_27a})^{A_27a}))\ c_2Emetric_2Emr1)\ V0g)) \\
& \quad V1f)) \Leftrightarrow (\exists V2k \in ty_2Erealax_2Ereal.(\exists V3N \in A_27a. \\
& \quad ((p\ (ap\ (ap\ V0g\ V3N)\ V3N)) \wedge (\forall V4n \in A_27a.((p\ (ap\ (ap\ V0g\ V4n) \\
& \quad V3N)) \Rightarrow (p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ (ap\ c_2Ereal_2Eabs\ (ap \\
& \quad V1f\ V4n))\ V2k))))))))) \\
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0g \in ((2^{A_27a})^{A_27a}). \\
& \quad (\forall V1x \in (ty_2Erealax_2Ereal^{A_27a}).(\forall V2x0 \in ty_2Erealax_2Ereal. \\
& \quad ((p\ (ap\ (ap\ (ap\ (c_2Enets_2Etends\ ty_2Erealax_2Ereal\ A_27a)\ V1x) \\
& \quad V2x0)\ (ap\ (ap\ (c_2Epair_2E_2C\ (ty_2Etopology_2Etopology\ ty_2Erealax_2Ereal) \\
& \quad ((2^{A_27a})^{A_27a}))\ (ap\ (c_2Emetric_2Emtop\ ty_2Erealax_2Ereal) \\
& \quad c_2Emetric_2Emr1))\ V0g))) \Leftrightarrow (p\ (ap\ (ap\ (ap\ (c_2Enets_2Etends\ ty_2Erealax_2Ereal \\
& \quad A_27a)\ (\lambda V3n \in A_27a.(ap\ (ap\ c_2Ereal_2Ereal_sub\ (ap\ V1x\ V3n)) \\
& \quad V2x0)))\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad (ty_2Etopology_2Etopology\ ty_2Erealax_2Ereal)\ ((2^{A_27a})^{A_27a})) \\
& \quad (ap\ (c_2Emetric_2Emtop\ ty_2Erealax_2Ereal)\ c_2Emetric_2Emr1)) \\
& \quad V0g)))))) \\
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0g \in ((2^{A.27a})^{A.27a}). \\
& (\forall V1x \in (ty_2Erealax_2Ereal^{A.27a}). (\forall V2x0 \in ty_2Erealax_2Ereal. \\
& (((p\ (ap\ (ap\ (ap\ (c_2Enets_2Etends\ ty_2Erealax_2Ereal\ A.27a)\ V1x) \\
V2x0)\ (ap\ (ap\ (c_2Epair_2E_2C\ (ty_2Etopology_2Etopology\ ty_2Erealax_2Ereal) \\
& ((2^{A.27a})^{A.27a}))\ (ap\ (c_2Emetric_2Emtop\ ty_2Erealax_2Ereal) \\
c_2Emetric_2Emr1))\ V0g))) \wedge (\neg (V2x0 = (ap\ c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)))) \Rightarrow (\exists V3N \in A.27a. ((p\ (ap\ (ap\ V0g\ V3N)\ V3N)) \wedge \\
& (\forall V4n \in A.27a. ((p\ (ap\ (ap\ V0g\ V4n)\ V3N)) \Rightarrow (\neg ((ap\ V1x\ V4n) = (\\
& ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))))))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0g \in ((2^{A.27a})^{A.27a}). \\
& (\forall V1x \in (ty_2Erealax_2Ereal^{A.27a}). (\forall V2x0 \in ty_2Erealax_2Ereal. \\
& (((p\ (ap\ (ap\ (ap\ (c_2Enets_2Etends\ ty_2Erealax_2Ereal\ A.27a)\ V1x) \\
V2x0)\ (ap\ (ap\ (c_2Epair_2E_2C\ (ty_2Etopology_2Etopology\ ty_2Erealax_2Ereal) \\
& ((2^{A.27a})^{A.27a}))\ (ap\ (c_2Emetric_2Emtop\ ty_2Erealax_2Ereal) \\
c_2Emetric_2Emr1))\ V0g))) \wedge (\neg (V2x0 = (ap\ c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)))) \Rightarrow (p\ (ap\ (ap\ (c_2Enets_2Ebounded\ ty_2Erealax_2Ereal \\
A.27a)\ (ap\ (ap\ (c_2Epair_2E_2C\ (ty_2Emetric_2Emetric\ ty_2Erealax_2Ereal) \\
& ((2^{A.27a})^{A.27a}))\ c_2Emetric_2Emr1)\ V0g))\ (\lambda V3n \in A.27a. (\\
& ap\ c_2Erealax_2Einv\ (ap\ V1x\ V3n))))))
\end{aligned} \tag{47}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n) \\
(ap\ c_2Enum_2ESUC\ V0n)))) \tag{48}$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
((ap\ (ap\ c_2Erealax_2Ereal_mul\ V0x)\ V1y) = (ap\ (ap\ c_2Erealax_2Ereal_mul \\
V1y)\ V0x)))) \tag{49}$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
(\forall V2z \in ty_2Erealax_2Ereal. ((ap\ (ap\ c_2Erealax_2Ereal_mul \\
V0x)\ (ap\ (ap\ c_2Erealax_2Ereal_mul\ V1y)\ V2z)) = (ap\ (ap\ c_2Erealax_2Ereal_mul \\
(ap\ (ap\ c_2Erealax_2Ereal_mul\ V0x)\ V1y))\ V2z)))))) \tag{50}$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. ((\neg (V0x = (ap\ c_2Ereal_2Ereal_of_num \\
c_2Enum_2E0))) \Rightarrow ((ap\ (ap\ c_2Erealax_2Ereal_mul\ (ap\ c_2Erealax_2Einv \\
V0x))\ V0x) = (ap\ c_2Ereal_2Ereal_of_num\ (ap\ c_2Earithmetic_2ENUMERAL \\
(ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))) \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) V0x)) \wedge (p (ap (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) V1y))) \Rightarrow (p (ap (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) (ap (ap c_2Erealax_2Ereal_mul V0x) V1y))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& V0x) (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) = V0x))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((\neg(V0x = (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0))) \Rightarrow ((ap (ap c_2Erealax_2Ereal_mul V0x) (ap c_2Erealax_2Einv \\
& V0x)) = (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((p (ap (ap c_2Ereal_2Ereal_lte \\
& V0x) V1y)) \wedge (p (ap (ap (ap c_2Erealax_2Ereal_lt V1y) V2z))) \Rightarrow (p (ap \\
& (ap c_2Erealax_2Ereal_lt V0x) V2z))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& V0x) (ap (ap c_2Ereal_2Ereal_sub V1y) V2z)) = (ap (ap c_2Ereal_2Ereal_sub \\
& (ap (ap c_2Erealax_2Ereal_mul V0x) V1y)) (ap (ap c_2Erealax_2Ereal_mul \\
& V0x) V2z))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((\neg(V0x = (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0))) \Rightarrow (\neg((ap c_2Erealax_2Einv V0x) = (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Erealax_2Ereal_lt V0x) V1y)) \Rightarrow (\neg(V0x = V1y))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((p (ap (ap c_2Erealax_2Ereal_lt \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x)) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) (ap c_2Erealax_2Einv \\
& V0x))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((p (ap (ap c_2Erealax_2Ereal_lt \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x)) \Rightarrow ((p (ap (ap \\
& c_2Erealax_2Ereal_lt (ap (ap c_2Erealax_2Ereal_mul V0x) V1y)) \\
& (ap (ap c_2Erealax_2Ereal_mul V0x) V2z))) \Leftrightarrow (p (ap (ap c_2Erealax_2Ereal_lt \\
& V1y) V2z))))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& V0m)) (ap c_2Ereal_2Ereal_of_num V1n))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0m) V1n))))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Ereal_2Ereal_sub \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = (ap c_2Erealax_2Ereal_neg \\
& V0x)))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap c_2Ereal_2Eabs (ap c_2Erealax_2Ereal_neg \\
& V0x)) = (ap c_2Ereal_2Eabs V0x)))
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (p (ap (ap c_2Ereal_2Ereal_lte \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) (ap c_2Ereal_2Eabs \\
& V0x))))
\end{aligned} \tag{64}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap c_2Ereal_2Eabs (ap (ap c_2Erealax_2Ereal_mul V0x) V1y)) = \\
& (ap (ap c_2Erealax_2Ereal_mul (ap c_2Ereal_2Eabs V0x)) (ap c_2Ereal_2Eabs \\
& V1y))))))
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
& (\forall V0w \in ty_2Erealax_2Ereal. (\forall V1x \in ty_2Erealax_2Ereal. \\
& (\forall V2y \in ty_2Erealax_2Ereal. (\forall V3z \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Eabs V0w)) V2y)) \wedge \\
& (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Eabs V1x)) V3z))) \Rightarrow \\
& (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Eabs (ap (ap c_2Erealax_2Ereal_mul \\
& V0w) V1x))) (ap (ap c_2Erealax_2Ereal_mul V2y) V3z)))))))))
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap c_2Ereal_2Eabs (ap (ap c_2Ereal_2Ereal_sub V0x) V1y)) = (\\
& ap c_2Ereal_2Eabs (ap (ap c_2Ereal_2Ereal_sub V1y) V0x))))))
\end{aligned} \tag{67}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((\neg(V0x = (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0))) \Leftrightarrow (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) (ap c_2Ereal_2Eabs V0x))))))
\end{aligned} \tag{68}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0g \in ((2^{A_27a})^{A_27a}). \\
& ((p (ap (c_2Enets_2Edorder A_27a) V0g)) \Rightarrow (\forall V1x \in (ty_2Erealax_2Ereal^{A_27a}). \\
& (\forall V2x0 \in ty_2Erealax_2Ereal. (((p (ap (ap (ap (c_2Enets_2Etends \\
& ty_2Erealax_2Ereal A_27a) V1x) V2x0) (ap (ap (c_2Epair_2E_2C (\\
& ty_2Etopology_2Etopology ty_2Erealax_2Ereal) ((2^{A_27a})^{A_27a})) \\
& (ap (c_2Emetric_2Emtop ty_2Erealax_2Ereal) c_2Emetric_2Emr1)) \\
& V0g))) \wedge (\neg(V2x0 = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)))))) \Rightarrow \\
& (p (ap (ap (ap (c_2Enets_2Etends ty_2Erealax_2Ereal A_27a) (\lambda V3n \in \\
& A_27a.(ap c_2Erealax_2Einv (ap V1x V3n)))) (ap c_2Erealax_2Einv \\
& V2x0)) (ap (ap (c_2Epair_2E_2C (ty_2Etopology_2Etopology ty_2Erealax_2Ereal) \\
& ((2^{A_27a})^{A_27a})) (ap (c_2Emetric_2Emtop ty_2Erealax_2Ereal) \\
& c_2Emetric_2Emr1)) V0g)))))))))
\end{aligned}$$