

thm_2Enets_2ENET__LE
(TMWkKP9prHSvY2DZjDsFXNHDCmb3qDGNUPu)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_21$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \tag{4}$$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x)) \mathbf{then} (the (\lambda x.x \in A \wedge p))$ of type $\iota \Rightarrow \iota$.

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})))$

Definition 5 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (ty$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal}) \tag{5}$$

Let $c_2Erealax_2Etreall_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (6)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (7)$$

Definition 6 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 7 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_ABS)$

Let $c_2Erealax_2Etreall_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (8)$$

Definition 8 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 9 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (9)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (10)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (11)$$

Definition 10 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{ty_2Enum_2Enum} \quad (12)$$

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (13)$$

Definition 11 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 12 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 13 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 14 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E$

Definition 15 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 16 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Definition 17 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 18 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap (ap (ap (c_2Ebool_2ECOND$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND \\ A_27a A_27b \in (A_27b)^{(ty_2Epair_2Eprod A_27a A_27b)} \end{aligned} \quad (14)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST \\ A_27a A_27b \in (A_27a)^{(ty_2Epair_2Eprod A_27a A_27b)} \end{aligned} \quad (15)$$

Definition 19 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c)^{A_27a}$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Emetric_2Emetric A0) \quad (16)$$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow c_2Emetric_2Emetric A_27a \in ((ty_2Emetric_2Emetric \\ A_27a)^{(ty_2Erealax_2Ereal)^{(ty_2Epair_2Eprod A_27a A_27a)}}) \end{aligned} \quad (17)$$

Definition 20 We define $c_2Emetric_2Emr1$ to be $(ap (c_2Emetric_2Emetric ty_2Erealax_2Ereal) (ap (c$

Definition 21 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 22 We define $c_2Enets_2Edorder$ to be $\lambda A_27a : \iota.\lambda V0g \in ((2^{A_27a})^{A_27a}).(ap (c_2Ebool_2E_21 A$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emetric_2Edist A_27a \in ((ty_2Erealax_2Ereal)^{(ty_2Epair_2Eprod A_27a A_27a)} \quad (18)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (19)$$

Definition 23 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (20)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Etopology_2Etopology\ A.27a \in ((ty_2Etopology_2Etopology\ A.27a)^{(2^{(2^A-27a)})}) \quad (21)$$

Definition 24 We define $c_2Emetric_2Emtop$ to be $\lambda A.27a : \iota.\lambda V0m \in (ty_2Emetric_2Emetric\ A.27a).(ap$

Let $c_2Enets_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c_2Enets_2Etends\ A.27a\ A.27b \in (((2^{(ty_2Epair_2Eprod\ (ty_2Etopology_2Etopology\ A.27a)\ ((2^{A-27b})^{A-27b}))})_{A.27a})_{(A.27a)^{A-27b}})) \quad (22)$$

Definition 25 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (23)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (24)$$

Definition 26 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})_{ty_2Enum_2Enum}) \quad (25)$$

Definition 27 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 28 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Erealax_2Etreal_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_inv \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (26)$$

Definition 29 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_ABS$

Let $c_2Erealax_2Etreal_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (27)$$

Definition 30 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0t1 \in ty_2Erealax_2Ereal.\lambda V1t2 \in ty_2Erealax_2Ereal.$

Definition 31 We define $c_2Ereal_2E_2F$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Definition 32 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2.$

Assume the following.

$$True \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge \\ & ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\ & A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg (\\ & p V0t)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\ & ((ap (ap (c_2Emetric_2Edist ty_2Erealax_2Ereal) c_2Emetric_2Emr1) \\ & (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\ & V0x) V1y)) = (ap c_2Ereal_2Eabs (ap (ap c_2Ereal_2Ereal_sub V1y) \\ & V0x)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0g \in ((2^{A_27a})^{A_27a}). \\ & ((p (ap (c_2Enets_2Edorder A_27a) V0g)) \Rightarrow (\forall V1P \in (2^{A_27a}). \\ & (\forall V2Q \in (2^{A_27a}).(((\exists V3n \in A_27a.((p (ap (ap V0g V3n) \\ & V3n)) \wedge (\forall V4m \in A_27a.((p (ap (ap V0g V4m) V3n)) \Rightarrow (p (ap V1P V4m)))))) \wedge \\ & (\exists V5n \in A_27a.((p (ap (ap V0g V5n) V5n)) \wedge (\forall V6m \in A_27a. \\ & ((p (ap (ap V0g V6m) V5n)) \Rightarrow (p (ap V2Q V6m)))))) \Rightarrow (\exists V7n \in A_27a. \\ & ((p (ap (ap V0g V7n) V7n)) \wedge (\forall V8m \in A_27a.((p (ap (ap V0g V8m) \\ & V7n)) \Rightarrow ((p (ap V1P V8m)) \wedge (p (ap V2Q V8m)))))))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0d \in (ty_2Emetric_2Emetric\ A_27a).(\forall V1g \in ((2^{A_27b})^{A_27b}). \\
& (\forall V2x \in (A_27a^{A_27b}).(\forall V3x0 \in A_27a.((p\ (ap\ (ap\ (ap \\
& (c_2Enets_2Etends\ A_27a\ A_27b)\ V2x)\ V3x0)\ (ap\ (ap\ (c_2Epair_2E_2C \\
& (ty_2Etopology_2Etopology\ A_27a)\ ((2^{A_27b})^{A_27b}))\ (ap\ (c_2Emetric_2Emtop \\
& A_27a)\ V0d))\ V1g))) \Leftrightarrow (\forall V4e \in ty_2Erealax_2Ereal.((p\ (ap \\
& (ap\ c_2Erealax_2Ereal_lt\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) \\
& V4e)) \Rightarrow (\exists V5n \in A_27b.((p\ (ap\ (ap\ V1g\ V5n)\ V5n)) \wedge (\forall V6m \in \\
& A_27b.((p\ (ap\ (ap\ V1g\ V6m)\ V5n)) \Rightarrow (p\ (ap\ (ap\ c_2Erealax_2Ereal_lt \\
& (ap\ (ap\ (c_2Emetric_2Edist\ A_27a)\ V0d)\ (ap\ (ap\ (c_2Epair_2E_2C \\
& A_27a\ A_27a)\ (ap\ V2x\ V6m))\ V3x0)))\ V4e))))))))) \\
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\
& ((\neg(p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ V0x)\ V1y))) \Leftrightarrow (p\ (ap\ (ap\ c_2Erealax_2Ereal_lt \\
& V1y)\ V0x)))) \\
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ (ap\ c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0))\ (ap\ (ap\ c_2Ereal_2Ereal_sub\ V0x)\ V1y))) \Leftrightarrow (p\ (ap \\
& (ap\ c_2Erealax_2Ereal_lt\ V1y)\ V0x)))) \\
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0d \in ty_2Erealax_2Ereal.((p\ (ap\ (ap\ c_2Erealax_2Ereal_lt \\
& (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))\ (ap\ (ap\ c_2Ereal_2E_2F \\
& V0d)\ (ap\ c_2Ereal_2Ereal_of_num\ (ap\ c_2Earithmetic_2ENUMERAL \\
& (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))))) \Leftrightarrow (p\ (\\
& ap\ (ap\ c_2Erealax_2Ereal_lt\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) \\
& V0d)))) \\
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap\ c_2Ereal_2Eabs\ (ap\ (ap\ c_2Ereal_2Ereal_sub\ V0x)\ V1y)) = (\\
& ap\ c_2Ereal_2Eabs\ (ap\ (ap\ c_2Ereal_2Ereal_sub\ V1y)\ V0x)))) \\
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x0 \in ty_2Erealax_2Ereal. (\forall V1x \in ty_2Erealax_2Ereal. \\
& (\forall V2y0 \in ty_2Erealax_2Ereal. (\forall V3y \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap c_2Erealax_2Ereal_lt V0x0) V2y0)) \wedge ((p (ap (ap c_2Erealax_2Ereal_lt \\
& (ap c_2Ereal_2Eabs (ap (ap c_2Ereal_2Ereal_sub V1x) V0x0))) (\\
& ap (ap c_2Ereal_2E_2F (ap (ap c_2Ereal_2Ereal_sub V2y0) V0x0)) \\
& (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmic_2ENUMERAL (\\
& ap c_2Earithmic_2EBIT2 c_2Earithmic_2EZERO)))))) \wedge (p (ap \\
& (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Eabs (ap (ap c_2Ereal_2Ereal_sub \\
& V3y) V2y0))) (ap (ap c_2Ereal_2E_2F (ap (ap c_2Ereal_2Ereal_sub \\
& V2y0) V0x0)) (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmic_2ENUMERAL \\
& (ap c_2Earithmic_2EBIT2 c_2Earithmic_2EZERO)))))) \Rightarrow (\\
& p (ap (ap c_2Erealax_2Ereal_lt V1x) V3y))))))
\end{aligned} \tag{41}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{42}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{45}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False))) \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(\\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{48}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0g \in ((2^{A_27a})^{A_27a}). \\ & ((p\ (ap\ (c_2Enets_2Edorder\ A_27a)\ V0g)) \Rightarrow (\forall V1x \in (ty_2Erealax_2Ereal^{A_27a}). \\ & (\forall V2x0 \in ty_2Erealax_2Ereal. (\forall V3y \in (ty_2Erealax_2Ereal^{A_27a}). \\ & (\forall V4y0 \in ty_2Erealax_2Ereal. (((p\ (ap\ (ap\ (ap\ (c_2Enets_2Etends \\ & ty_2Erealax_2Ereal\ A_27a)\ V1x)\ V2x0)\ (ap\ (ap\ (c_2Epair_2E_2C\ (\\ & ty_2Etopology_2Etopology\ ty_2Erealax_2Ereal)\ ((2^{A_27a})^{A_27a})) \\ & (ap\ (c_2Emetric_2Emtop\ ty_2Erealax_2Ereal)\ c_2Emetric_2Emr1)) \\ & V0g))) \wedge ((p\ (ap\ (ap\ (ap\ (c_2Enets_2Etends\ ty_2Erealax_2Ereal\ A_27a) \\ & V3y)\ V4y0)\ (ap\ (ap\ (c_2Epair_2E_2C\ (ty_2Etopology_2Etopology \\ & ty_2Erealax_2Ereal)\ ((2^{A_27a})^{A_27a}))\ (ap\ (c_2Emetric_2Emtop \\ & ty_2Erealax_2Ereal)\ c_2Emetric_2Emr1))\ V0g))) \wedge (\exists V5N \in \\ & A_27a. ((p\ (ap\ (ap\ V0g\ V5N)\ V5N)) \wedge (\forall V6n \in A_27a. ((p\ (ap\ (ap \\ & V0g\ V6n)\ V5N)) \Rightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ V1x\ V6n))\ (ap \\ & V3y\ V6n)))))))))) \Rightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ V2x0)\ V4y0))))))))) \end{aligned}$$