thm_2Enets_2ENET__NULL (TMRKCcUd-EvW9cjkNWzmLT8UKAnDgQQDdWg9)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{3}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{4}$$

Definition 3 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (\mathsf{c\_2Emin\_2E\_40}\ (ty$

Let $c\_2Erealax\_2Etreal\_\_neg : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{5}$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{6}$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehr} \tag{7}$$

Definition 6 *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_$

Definition 7 *We define* c_2Erealax_2Ereal__neg *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal__

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{8}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{9}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{10}$$

Definition 8 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{11}$$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{12}$$

Definition 9 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2$

Definition 10 *We define* c_2Ebool_2EF *to be* $(ap$ (c_2Ebool_2E_21 2) $(\lambda V0t \in 2.V0t))$.

Definition 11 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P {\Rightarrow} p\ Q)$ *of type* ι.

Definition 12 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 13 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Definition 14 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 15 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 16 *We define* c_2Ereal_2Eabs *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (ap$ (c_2Ebool_2ECONI

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \quad (13)$$

Definition 17 *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Definition 18 *We define* c_2Ereal_2Ereal__sub *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2E$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (14)$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (15)$$

Definition 19 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Let $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Emetric\_2Emetric\ A0) \quad (16)$$

Let $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Emetric\ A\_27a \in ((ty\_2Emetric\_2Emetric\ A\_27a)^{(ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})}) \quad (17)$$

Definition 20 *We define* c_2Emetric_2Emr1 *to be* $(ap\ (c\_2Emetric\_2Emetric\ ty\_2Erealax\_2Ereal)\ (ap$ (c_

Let $c\_2Emetric\_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Edist\ A\_27a \in ((ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a} \quad (18)$$

Definition 21 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \quad (19)$$

Definition 22 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $\mathit{ty\_2Etopology\_2Etopology} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\mathit{nonempty}\ A0 \Rightarrow \mathit{nonempty}\ (\mathit{ty\_2Etopology\_2Etopology}\ A0) \tag{20}$$

Let $\mathit{c\_2Etopology\_2Etopology} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.\mathit{nonempty}\ A\_27a \Rightarrow \mathit{c\_2Etopology\_2Etopology}\ A\_27a \in ((\mathit{ty\_2Etopology\_2Etopology}\ A\_27a)^{(2^{(2^{A\_27a})})}) \tag{21}$$

Definition 23 *We define* c_2Emetric_2Emtop *to be* $\lambda A\_27a : \iota.\lambda V0m \in (\mathit{ty\_2Emetric\_2Emetric}\ A\_27a).(ap$

Let $\mathit{c\_2Enets\_2Etends} : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.\mathit{nonempty}\ A\_27a \Rightarrow \forall A\_27b.\mathit{nonempty}\ A\_27b \Rightarrow \mathit{c\_2Enets\_2Etends}\ A\_27a\ A\_27b \in (((2^{(\mathit{ty\_2Epair\_2Eprod}\ (\mathit{ty\_2Etopology\_2Etopology}\ A\_27a)\ ((2^{A\_27b})^{A\_27b}))})^{A\_27a})^{(A\_27a^{A\_27b})}) \tag{22}$$

Assume the following.

$$\mathit{True} \tag{23}$$

Assume the following.

$$\forall A\_27a.\mathit{nonempty}\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \mathit{True})) \tag{24}$$

Assume the following.

$$(\forall V0x \in \mathit{ty\_2Erealax\_2Ereal}.(\forall V1y \in \mathit{ty\_2Erealax\_2Ereal}.((ap\ (ap\ (\mathit{c\_2Emetric\_2Edist}\ \mathit{ty\_2Erealax\_2Ereal})\ \mathsf{c\_2Emetric\_2Emr1})\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ \mathit{ty\_2Erealax\_2Ereal}\ \mathit{ty\_2Erealax\_2Ereal})\ V0x)\ V1y)) = (ap\ \mathsf{c\_2Ereal\_2Eabs}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ V1y)\ V0x))))) \tag{25}$$

Assume the following.

$$\forall A\_27a.\mathit{nonempty}\ A\_27a \Rightarrow \forall A\_27b.\mathit{nonempty}\ A\_27b \Rightarrow (\forall V0d \in (\mathit{ty\_2Emetric\_2Emetric}\ A\_27a).(\forall V1g \in ((2^{A\_27b})^{A\_27b}).(\forall V2x \in (A\_27a^{A\_27b}).(\forall V3x0 \in A\_27a.((p\ (ap\ (ap\ (ap\ (\mathit{c\_2Enets\_2Etends}\ A\_27a\ A\_27b)\ V2x)\ V3x0)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (\mathit{ty\_2Etopology\_2Etopology}\ A\_27a)\ ((2^{A\_27b})^{A\_27b}))\ (ap\ (\mathsf{c\_2Emetric\_2Emtop}\ A\_27a)\ V0d))\ V1g))) \Leftrightarrow (\forall V4e \in \mathit{ty\_2Erealax\_2Ereal}.((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ \mathit{c\_2Ereal\_2Ereal\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))\ V4e)) \Rightarrow (\exists V5n \in A\_27b.((p\ (ap\ (ap\ V1g\ V5n)\ V5n)) \wedge (\forall V6m \in A\_27b.((p\ (ap\ (ap\ V1g\ V6m)\ V5n)) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ (ap\ (\mathit{c\_2Emetric\_2Edist}\ A\_27a)\ V0d)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ (ap\ V2x\ V6m))\ V3x0)))\ V4e)))))))))))) \tag{26}$$

Assume the following.

$$(\forall V0x \in \mathit{ty\_2Erealax\_2Ereal}.(\forall V1y \in \mathit{ty\_2Erealax\_2Ereal}.((ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ V0x)\ V1y)) = (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ V1y)\ V0x)))) \tag{27}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V0x) = (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V0x))) \tag{28}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0g \in ((2^{A\_27a})^{A\_27a}).(\forall V1x \in (ty\_2Erealax\_2Ereal^{A\_27a}).(\forall V2x0 \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ (ap\ (c\_2Enets\_2Etends\ ty\_2Erealax\_2Ereal\ A\_27a)\ V1x)\ V2x0)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Etopology\_2Etopology\ ty\_2Erealax\_2Ereal)\ ((2^{A\_27a})^{A\_27a}))\ (ap\ (\mathsf{c\_2Emetric\_2Emtop}\ ty\_2Erealax\_2Ereal)\ \mathsf{c\_2Emetric\_2Emr1}))\ V0g))) \Leftrightarrow (p\ (ap\ (ap\ (ap\ (c\_2Enets\_2Etends\ ty\_2Erealax\_2Ereal\ A\_27a)\ (\lambda V3n \in A\_27a.(ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ (ap\ V1x\ V3n))\ V2x0)))\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Etopology\_2Etopology\ ty\_2Erealax\_2Ereal)\ ((2^{A\_27a})^{A\_27a}))\ (ap\ (\mathsf{c\_2Emetric\_2Emtop}\ ty\_2Erealax\_2Ereal)\ \mathsf{c\_2Emetric\_2Emr1}))\ V0g)))))))$$