

thm_2Enets_2ENET__NULL__CMUL
(TMSWbnCz1yuF5gps5iDbZDE35bL4v5mng8q)

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{6}$$

Definition 7 We define $c_2Earithmic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmic_2E$

Definition 8 We define $c_2Earithmic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 9 We define $c_2Emin_2E_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 10 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V0t)))$

Definition 11 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (7)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (8)$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (9)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (10)$$

Definition 12 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge P x)$) of type $\iota \Rightarrow \iota$.

Definition 13 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (t$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal}) \quad (11)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal}) \quad (12)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (13)$$

Definition 14 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 15 We define $c_Erealax_Ereal_neg$ to be $\lambda V0T1 \in ty_Erealax_Ereal.(ap\ c_Erealax_Ereal$
 Let $c_Erealax_Etrealm_add : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_add \in (((ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)\ (ty_Epair_Eprod\ ty_Ehreal_Ehreal)\ (ty_Epair_Eprod\ ty_Ehreal_Ehreal))\ (ty_Epair_Eprod\ ty_Ehreal_Ehreal))\ (ty_Epair_Eprod\ ty_Ehreal_Ehreal) \quad (14)$$

Definition 16 We define $c_Erealax_Ereal_add$ to be $\lambda V0T1 \in ty_Erealax_Ereal.\lambda V1T2 \in ty_Erealax_Ereal$

Definition 17 We define $c_Ereal_Ereal_sub$ to be $\lambda V0x \in ty_Erealax_Ereal.\lambda V1y \in ty_Erealax_Ereal$

Let $c_Ereal_Ereal_of_num : \iota$ be given. Assume the following.

$$c_Ereal_Ereal_of_num \in (ty_Erealax_Ereal^{ty_Eenum_Eenum}) \quad (15)$$

Let $c_Erealax_Etrealm_lt : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_lt \in ((2^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)}\ (ty_Epair_Eprod\ ty_Ehreal_Ehreal))\ (ty_Epair_Eprod\ ty_Ehreal_Ehreal)) \quad (16)$$

Definition 18 We define $c_Erealax_Ereal_lt$ to be $\lambda V0T1 \in ty_Erealax_Ereal.\lambda V1T2 \in ty_Erealax_Ereal$

Definition 19 We define c_Ebool_E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_Eemin_E3D_3D_3E\ V0t)\ c_Ebool_E7E$

Definition 20 We define $c_Ereal_Ereal_lte$ to be $\lambda V0x \in ty_Erealax_Ereal.\lambda V1y \in ty_Erealax_Ereal$

Definition 21 We define $c_Ebool_E2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_Ebool_E2E_21\ 2)\ (\lambda V2t \in$

Definition 22 We define $c_Ebool_E2ECOND$ to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 23 We define c_Ereal_E2Eabs to be $\lambda V0x \in ty_Erealax_Ereal.(ap\ (ap\ (ap\ (c_Ebool_E2ECOND$

Let $c_Epair_E2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Epair_E2ESND\ A_27a\ A_27b \in (A_27b^{(ty_Epair_E2Eprod\ A_27a\ A_27b)}) \quad (17)$$

Let $c_Epair_E2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Epair_E2EFST\ A_27a\ A_27b \in (A_27a^{(ty_Epair_E2Eprod\ A_27a\ A_27b)}) \quad (18)$$

Definition 24 We define $c_Epair_E2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27$

Let $ty_E2Emetric_E2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_E2Emetric_E2Emetric\ A0) \quad (19)$$

Let $c_E2Emetric_E2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_E2Emetric_E2Emetric\ A_27a \in ((ty_E2Emetric_E2Emetric\ A_27a)^{(ty_Erealax_Ereal^{(ty_Epair_E2Eprod\ A_27a\ A_27a)})}) \quad (20)$$

Definition 25 We define $c_2Emetric_2Emr1$ to be $(ap (c_2Emetric_2Emetric\ ty_2Erealx_2Ereal) (ap (c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Edist\ A_27a \in ((ty_2Erealx_2Ereal^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})^{(2^{A-27a})}) \quad (21)$$

Definition 26 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$ Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A-27b})^{A-27a}}) \quad (22)$$

Definition 27 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2E$ Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (23)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopology\ A_27a \in ((ty_2Etopology_2Etopology\ A_27a)^{(2^{(2^{A-27a})})}) \quad (24)$$

Definition 28 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota. \lambda V0m \in (ty_2Emetric_2Emetric\ A_27a). (ap$

Let $c_2Enets_2Eextends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Enets_2Eextends\ A_27a\ A_27b \in (((2^{(ty_2Epair_2Eprod\ (ty_2Etopology_2Etopology\ A_27a)\ ((2^{A-27b})^{A-27b}))})_{A_27a})^{(A_27a^{A-27b})}) \quad (25)$$

Let $c_2Erealx_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealx_2Etrealmul \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (26)$$

Definition 29 We define $c_2Erealx_2Einv$ to be $\lambda V0T1 \in ty_2Erealx_2Ereal. (ap\ c_2Erealx_2Ereal_ABS$

Definition 30 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Let $c_2Erealx_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealx_2Etrealmul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (27)$$

Definition 31 We define $c_2Erealx_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealx_2Ereal. \lambda V1T2 \in ty_2Erealx$

Definition 32 We define $c_Ereal_2E_2F$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Assume the following.

$$\begin{aligned} ((ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)) = \\ (ap\ c_2Enum_2ESUC\ c_2Enum_2E0)) = \end{aligned} \quad (28)$$

Assume the following.

$$True \quad (29)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (30)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (31)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (33)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} ((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (36)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (37)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\
& p V0t))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in \\
& A_27a. (((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\
& V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF \\
& V0t1) V1t2) = V1t2))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap (ap (c_2Emetric_2Edist ty_2Erealax_2Ereal) c_2Emetric_2Emr1) \\
& (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& V0x) V1y)) = (ap c_2Ereal_2Eabs (ap (ap c_2Ereal_2Ereal_sub V1y) \\
& V0x))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\
& \forall V0d \in (ty_2Emetric_2Emetric A_27a). (\forall V1g \in ((2^{A_27b})^{A_27b}). \\
& (\forall V2x \in (A_27a^{A_27b}). (\forall V3x0 \in A_27a. ((p (ap (ap (ap \\
& (c_2Enets_2Etends A_27a A_27b) V2x) V3x0) (ap (ap (c_2Epair_2E_2C \\
& (ty_2Etopology_2Etopology A_27a) ((2^{A_27b})^{A_27b})) (ap (c_2Emetric_2Emtop \\
& A_27a) V0d)) V1g))) \Leftrightarrow (\forall V4e \in ty_2Erealax_2Ereal. ((p (ap \\
& (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \\
& V4e)) \Rightarrow (\exists V5n \in A_27b. ((p (ap (ap V1g V5n) V5n)) \wedge (\forall V6m \in \\
& A_27b. ((p (ap (ap V1g V6m) V5n)) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt \\
& (ap (ap (c_2Emetric_2Edist A_27a) V0d) (ap (ap (c_2Epair_2E_2C \\
& A_27a A_27a) (ap V2x V6m)) V3x0))) V4e))))))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (p (ap (ap c_2Eprim_rec_2E_3C V0n) \\
& (ap c_2Enum_2ESUC V0n))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) V0x)) \wedge (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) V1y))) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) (ap (ap c_2Erealax_2Ereal_mul V0x) V1y))))))
\end{aligned} \tag{43}$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_mul (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))) \quad (44)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(p (ap (ap c_2Ereal_2Ereal_lte V0x) V0x))) \quad (45)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x)) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) (ap c_2Erealax_2Einv V0x)))))) \quad (46)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. (\forall V2z \in ty_2Erealax_2Ereal.((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x)) \Rightarrow ((p (ap (ap c_2Erealax_2Ereal_lt (ap (ap c_2Erealax_2Ereal_mul V0x) V1y)) (ap (ap c_2Erealax_2Ereal_mul V0x) V2z))) \Leftrightarrow (p (ap (ap c_2Erealax_2Ereal_lt V1y) V2z)))))))) \quad (47)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num V0m)) (ap c_2Ereal_2Ereal_of_num V1n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)))))) \quad (48)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. ((\neg (V1y = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))) \Rightarrow ((ap (ap c_2Erealax_2Ereal_mul V1y) (ap (ap c_2Ereal_2E_2F V0x) V1y)) = V0x)))) \quad (49)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Ereal_2Ereal_sub (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = (ap c_2Erealax_2Ereal_neg V0x))) \quad (50)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.(((ap\ c_2Ereal_2Eabs\ V0x) = \\
& (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) \Leftrightarrow (V0x = (ap\ c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.((ap\ c_2Ereal_2Eabs\ (ap\ c_2Erealax_2Ereal_neg \\
& \quad V0x)) = (ap\ c_2Ereal_2Eabs\ V0x)))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap\ c_2Ereal_2Eabs\ (ap\ (ap\ c_2Erealax_2Ereal_mul\ V0x)\ V1y)) = \\
& (ap\ (ap\ c_2Erealax_2Ereal_mul\ (ap\ c_2Ereal_2Eabs\ V0x))\ (ap\ c_2Ereal_2Eabs \\
& \quad V1y))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.((\neg(V0x = (ap\ c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0))) \Leftrightarrow (p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ (ap\ c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0))\ (ap\ c_2Ereal_2Eabs\ V0x))))))
\end{aligned} \tag{54}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0g \in ((2^{A_27a})^{A_27a}). \\
& (\forall V1k \in ty_2Erealax_2Ereal.(\forall V2x \in (ty_2Erealax_2Ereal^{A_27a}). \\
& ((p\ (ap\ (ap\ (ap\ (c_2Enets_2Eextends\ ty_2Erealax_2Ereal\ A_27a)\ V2x) \\
& (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad (ty_2Etopology_2Etopology\ ty_2Erealax_2Ereal)\ ((2^{A_27a})^{A_27a})) \\
& \quad (ap\ (c_2Emetric_2Emtop\ ty_2Erealax_2Ereal)\ c_2Emetric_2Emr1)) \\
& V0g))) \Rightarrow (p\ (ap\ (ap\ (ap\ (c_2Enets_2Eextends\ ty_2Erealax_2Ereal\ A_27a) \\
& (\lambda V3n \in A_27a.(ap\ (ap\ c_2Erealax_2Ereal_mul\ V1k)\ (ap\ V2x\ V3n)))) \\
& (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad (ty_2Etopology_2Etopology\ ty_2Erealax_2Ereal)\ ((2^{A_27a})^{A_27a})) \\
& \quad (ap\ (c_2Emetric_2Emtop\ ty_2Erealax_2Ereal)\ c_2Emetric_2Emr1)) \\
& \quad V0g))))))
\end{aligned}$$