

thm_2Enets_2ESEQ__TENDS
(TMGCL8ShHrMs75hTYovX2Qt73jZhRDCMtZN)

October 26, 2020

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_F$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num (ap c_2Enum_2EREP_num (ap c_2Enum_2ESUC_REP V0m)))$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p x)$) of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_Emin_2E_40$

Definition 11 We define $c_Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 13 We define $c_Earithmic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 14 We define $c_Earithmic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 15 We define $c_Earithmic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (5)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (6)$$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_2Emetric_2Emetric\ A0) \quad (7)$$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Emetric_2Edist\ A_27a \in ((ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}) \quad (8)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (9)$$

Definition 16 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (10)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (11)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (12)$$

Definition 17 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal. (ap\ (c_Emin_2E_40\ (t$

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (13)$$

Definition 18 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (14)$$

Definition 19 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ V0x\ V1y)$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (15)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopology\ A_27a \in ((ty_2Etopology_2Etopology\ A_27a)^{(2^{(2^{A_27a})})}) \quad (16)$$

Definition 20 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Emetric_2Emetric\ A_27a).(ap\ (c_2Epair_2E_2C\ A_27a\ A_27a)\ V0m\ V0m)$

Let $c_2Enets_2Eetends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Enets_2Eetends\ A_27a\ A_27b \in (((2^{(ty_2Epair_2Eprod\ (ty_2Etopology_2Etopology\ A_27a)\ (2^{A_27b})^{A_27b}))})^{A_27a})^{(A_27a^{A_27b})}) \quad (17)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V0m)\ V0m))) \quad (18)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\forall V1m \in ty_2Enum_2Enum.(p\ (ap\ (ap\ c_2Earithmetic_2E_3E_3D\ V0n)\ V1m)) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V1m)\ V0n)))) \quad (19)$$

Assume the following.

$$True \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (21)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0d \in (ty_2Emetric_2Emetric\ A.27a).(\forall V1g \in ((2^{A.27b})^{A.27b}). \\ & (\forall V2x \in (A.27a^{A.27b}).(\forall V3x0 \in A.27a.((p\ (ap\ (ap\ (ap \\ & (c_2Enets_2Etends\ A.27a\ A.27b)\ V2x)\ V3x0)\ (ap\ (ap\ (c_2Epair_2E_2C \\ & (ty_2Etopology_2Etopology\ A.27a)\ ((2^{A.27b})^{A.27b}))\ (ap\ (c_2Emetric_2Emtop \\ & A.27a)\ V0d))\ V1g))) \Leftrightarrow (\forall V4e \in ty_2Erealax_2Ereal.((p\ (ap \\ & (ap\ c_2Erealax_2Ereal_lt\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) \\ & V4e)) \Rightarrow (\exists V5n \in A.27b.((p\ (ap\ (ap\ V1g\ V5n)\ V5n)) \wedge (\forall V6m \in \\ & A.27b.((p\ (ap\ (ap\ V1g\ V6m)\ V5n)) \Rightarrow (p\ (ap\ (ap\ c_2Erealax_2Ereal_lt \\ & (ap\ (ap\ (c_2Emetric_2Edist\ A.27a)\ V0d)\ (ap\ (ap\ (c_2Epair_2E_2C \\ & A.27a\ A.27a)\ (ap\ V2x\ V6m))\ V3x0)))\ V4e))))))))) \end{aligned} \quad (24)$$

Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0d \in (ty_2Emetric_2Emetric \\ & A.27a).(\forall V1x \in (A.27a^{ty_2Enum_2Enum}).(\forall V2x0 \in A.27a. \\ & ((p\ (ap\ (ap\ (ap\ (c_2Enets_2Etends\ A.27a\ ty_2Enum_2Enum)\ V1x)\ V2x0) \\ & (ap\ (ap\ (c_2Epair_2E_2C\ (ty_2Etopology_2Etopology\ A.27a)\ ((2^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})) \\ & (ap\ (c_2Emetric_2Emtop\ A.27a)\ V0d))\ c_2Earithmetic_2E_3E_3D))) \Leftrightarrow \\ & (\forall V3e \in ty_2Erealax_2Ereal.((p\ (ap\ (ap\ c_2Erealax_2Ereal_lt \\ & (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))\ V3e)) \Rightarrow (\exists V4N \in \\ & ty_2Enum_2Enum.(\forall V5n \in ty_2Enum_2Enum.((p\ (ap\ (ap\ c_2Earithmetic_2E_3E_3D \\ & V5n)\ V4N)) \Rightarrow (p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ (ap\ (ap\ (c_2Emetric_2Edist \\ & A.27a)\ V0d)\ (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27a)\ (ap\ V1x\ V5n))\ V2x0))) \\ & V3e))))))))) \end{aligned}$$