thm_2EnumRing_2Enum__ring__thms (TMHKZe4uPKU68pKDH29vtBugqR85ybxcNDe)

October 26, 2020

Let $ty\_2EternaryComparisons\_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty\_2EternaryComparisons\_2Eordering \tag{1}$$

Let $ty\_2Equote\_2Eindex : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Equote\_2Eindex \tag{2}$$

Let $c\_2Equote\_2Eindex\_\_compare : \iota$ be given. Assume the following.

$$c\_2Equote\_2Eindex\_\_compare \in ((ty\_2EternaryComparisons\_2Eordering^{ty\_2Equote\_2Eindex})^{ty\_2Equote\_2E} \tag{3}$$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{4}$$

Let $c\_2EternaryComparisons\_2Elist\_\_compare : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2EternaryComparisons$$
$$A\_27a\ A\_27b \in (((ty\_2EternaryComparisons\_2Eordering^{(ty\_2Elist\_2Elist\ A\_27b)})^{(ty\_2Elist\_2Elist\ A\_27a)})^{((ty} \tag{5}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{6}$$

Let $c\_2EternaryComparisons\_2Eordering2num : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2Eordering2num \in (ty\_2Enum\_2Enum^{ty\_2EternaryComparisons\_2Eordering}) \tag{7}$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{8}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{9}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 3 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{10}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{11}$$

Definition 4 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 6 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{12}$$

Definition 7 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 8 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Definition 9 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 10 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 11 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 12 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge$ *of type* $\iota \Rightarrow \iota$.

Definition 13 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 14 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Definition 15 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 16 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 17 *We define* c_2EternaryComparisons_2Eordering__CASE *to be* $\lambda A\_27a : \iota.\lambda V0x \in ty\_2Eternary$

Let $c\_2EternaryComparisons\_2ELESS : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2ELESS \in ty\_2EternaryComparisons\_2Eordering \tag{13}$$

Definition 18 *We define* c_2Equote_2Eindex__lt *to be* $\lambda V0i1 \in ty\_2Equote\_2Eindex.\lambda V1i2 \in ty\_2Equote\_2$

Let $c\_2EternaryComparisons\_2Elist\_\_merge : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2EternaryComparisons\_2Elist\_\_merge\ A\_27a \in ((((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)})^{((2^{A\_27a})^{A\_27a})}) \tag{14}$$

Let $ty\_2Ecanonical\_2Ecanonical\_\_sum : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A0) \tag{15}$$

Let $c\_2Ecanonical\_2ECons\_\_monom : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ECons\_\_monom\ A\_27a \in ((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2Elist\ ty\_2Eq} \tag{16}$$

Let $c\_2Ecanonical\_2ENil\_\_monom : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ENil\_\_monom\ A\_27a \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a) \tag{17}$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{18}$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{19}$$

Let $c\_2Ecanonical\_2ECons\_\_varlist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a \in (((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2Eli} \tag{20}$$

Let $ty\_2Ecanonical\_2Espolynom : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ecanonical\_2Espolynom\ A0) \tag{21}$$

Let $c\_2Ecanonical\_2ESPmult : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ESPmult\ A\_27a \in (((ty\_2Ecanonical\_2Espolynom\ A\_27a)^{(ty\_2Ecanonical\_2Espolynom\ A\_27a)})^{(ty\_2Ecanonical\_2Espolynom\ A\_27a)}) \tag{22}$$

Let $c\_2Ecanonical\_2ESPplus : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ESPplus\ A\_27a \in (((ty\_2Ecanonical\_2Espolynom\ A\_27a)^{(ty\_2Ecanonical\_2Espolynom\ A\_27a)})^{(ty\_2Ecanonical\_2Espolynom\ A\_27a)}) \tag{23}$$

Let $c\_2Ecanonical\_2ESPvar : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ESPvar\ A\_27a \in ((ty\_2Ecanonical\_2Espolynom\ A\_27a)^{ty\_2Equote\_2Eindex}) \tag{24}$$

Let $c\_2Ecanonical\_2ESPconst : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ESPconst\ A\_27a \in ((ty\_2Ecanonical\_2Espolynom\ A\_27a)^{A\_27a}) \tag{25}$$

Let $ty\_2Esemi\_\_ring\_2Esemi\_\_ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A0) \tag{26}$$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{27}$$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{28}$$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR0\ A\_27a \in (A\_27a^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{29}$$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1\ A\_27a \in (A\_27a^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{30}$$

Definition 19 *We define* c_2Esemi__ring_2Eis__semi__ring *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Esemi\_\_ring\_2Esen$

Let $ty\_2Equote\_2Evarmap : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Equote\_2Evarmap\ A0) \tag{31}$$

Let $c\_2Ecanonical\_2Einterp\_\_sp : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Einterp\_\_sp\ A\_27a \in (((A\_27a^{(ty\_2Ecanonical\_2Espolynom\ A\_27a)})^{(ty\_2Equote\_2Evarmap\ A\_27a)})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{32}$$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (33)$$

Let $c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a \in (((((ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)^{((A\_27a^{A\_27a})^{A\_27a})})^{((A\_27a^{A\_27a})^{A\_27a})})^{A\_27a})^{A\_27a}) \quad (34)$$

Definition 20 *We define* c_2EnumRing_2Enum__interp__sp *to be* $(ap\ (c\_2Ecanonical\_2Einterp\_\_sp\ ty\_2Enu$

Let $c\_2Ecanonical\_2Espolynom\_\_normalize : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Espolynom\_\_normalize\ A\_27a \in (((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Espolynom\ A\_27a)})^{(ty\_2Esemi\_\_ring\_2E} \quad (35)$$

Let $c\_2Ecanonical\_2Ecanonical\_\_sum\_\_simplify : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Ecanonical\_\_sum\_\_simplify\ A\_27a \in (((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Esemi\_\_ri} \quad (36)$$

Definition 21 *We define* c_2Ecanonical_2Espolynom__simplify *to be* $\lambda A\_27a : \iota.\lambda V0sr \in (ty\_2Esemi\_\_ring\_2$

Definition 22 *We define* c_2EnumRing_2Enum__spolynom__simplify *to be* $(ap$ (c_2Ecanonical_2Espolynom__si

Definition 23 *We define* c_2EnumRing_2Enum__spolynom__normalize *to be* $(ap\ (c\_2Ecanonical\_2Espolynom$

Let $c\_2Ecanonical\_2Einterp\_\_cs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Einterp\_\_cs\ A\_27a \in (((A\_27a^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Equote\_2Evarmap\ A\_27a)})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_2} \quad (37)$$

Definition 24 *We define* c_2EnumRing_2Enum__interp__cs *to be* $(ap\ (c\_2Ecanonical\_2Einterp\_\_cs\ ty\_2Enu$

Let $c\_2Ecanonical\_2Eics\_\_aux : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Eics\_\_aux\ A\_27a \in ((((A\_27a^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{A\_27a})^{(ty\_2Equote\_2Evarmap\ A\_27a)})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_r} \quad (38)$$

Definition 25 *We define* c_2EnumRing_2Enum__ics__aux *to be* $(ap\ (c\_2Ecanonical\_2Eics\_\_aux\ ty\_2Enum\_2$

Let $c\_2Ecanonical\_2Einterp\_\_m : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Einterp\_\_m\ A\_27a \in ((((A\_27a^{(ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex)})^{A\_27a})^{(ty\_2Equote\_2Evarmap\ A\_27a)})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring} \quad (39)$$

Definition 26 *We define* c_2EnumRing_2Enum__interp__m *to be* $(ap\ (c\_2Ecanonical\_2Einterp\_\_m\ ty\_2Enu$

Let $c\_2Ecanonical\_2Einterp\_\_vl : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Einterp\_\_vl\ A\_27a \in (((A\_27a^{(ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex)})^{(ty\_2Equote\_2Evarmap\ A\_27a)})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \quad (40)$$

Definition 27 *We define* c_2EnumRing_2Enum__interp__vl *to be* $(ap\ (c\_2Ecanonical\_2Einterp\_\_vl\ ty\_2Enu$

Let $c\_2Ecanonical\_2Eivl\_\_aux : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Eivl\_\_aux\ A\_27a \in ((((A\_27a^{(ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex)})^{ty\_2Equote\_2Eindex})^{(ty\_2Equote\_2Evarmap\ A\_27a)})^{(ty\_2Esemi\_\_ring\_2} \quad (41)$$

Definition 28 *We define* c_2EnumRing_2Enum__ivl__aux *to be* $(ap\ (c\_2Ecanonical\_2Eivl\_\_aux\ ty\_2Enum\_2$

Definition 29 *We define* c_2EnumRing_2Enum__canonical__sum__simplify *to be* $(ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_simplify\ ty\_2Enum\_2Enum)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2$

Let $c\_2Ecanonical\_2Ecanonical\_\_sum\_\_prod : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Ecanonical\_\ A\_27a \in ((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Ecanonic} \quad (42)$$

Definition 30 *We define* c_2EnumRing_2Enum__canonical__sum__prod *to be* $(ap\ (c\_2Ecanonical\_2Ecanonic$

Let $c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar3 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Ecanonical\_\ A\_27a \in (((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2E} \quad (43)$$

Definition 31 *We define* c_2EnumRing_2Enum__canonical__sum__scalar3 *to be* $(ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar3\ ty\_2Enum\_2Enum)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2E$

Let $c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar2 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Ecanonical\_\_sun\ A\_27a \in ((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2El} \quad (44)$$

Definition 32 *We define* c_2EnumRing_2Enum__canonical__sum__scalar2 *to be* $(ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar2\ ty\_2Enum\_2Enum)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2E$

Let $c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar\ A\_27a \in ((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{A\_27a})^{(ty\_2Es} \tag{45}$$

Definition 33 *We define* c_2EnumRing_2Enum__canonical__sum__scalar *to be* $(ap\ (c\_2Ecanonical\_2Ecanoni$

Let $c\_2Ecanonical\_2Evarlist\_\_insert : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Evarlist\_\_i\ A\_27a \in ((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2El} \tag{46}$$

Definition 34 *We define* c_2EnumRing_2Enum__varlist__insert *to be* $(ap\ (c\_2Ecanonical\_2Evarlist\_\_insert$

Let $c\_2Ecanonical\_2Emonom\_\_insert : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Emonon\ A\_27a \in (((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2E} \tag{47}$$

Definition 35 *We define* c_2EnumRing_2Enum__monom__insert *to be* $(ap\ (c\_2Ecanonical\_2Emonom\_\_inser$

Let $c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Ecanonical\_\_s\ A\_27a \in ((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Ecanonic} \tag{48}$$

Definition 36 *We define* c_2EnumRing_2Enum__canonical__sum__merge *to be* $(ap\ (c\_2Ecanonical\_2Ecanoni$

Let $c\_2Equote\_2EEmpty\_\_vm : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Equote\_2EEmpty\_\_vm\ A\_27a \in (ty\_2Equote\_2Evarmap\ A\_27a) \tag{49}$$

Let $c\_2Equote\_2ELeft\_\_idx : \iota$ be given. Assume the following.

$$c\_2Equote\_2ELeft\_\_idx \in (ty\_2Equote\_2Eindex^{ty\_2Equote\_2Eindex}) \tag{50}$$

Let $c\_2Equote\_2ERight\_\_idx : \iota$ be given. Assume the following.

$$c\_2Equote\_2ERight\_\_idx \in (ty\_2Equote\_2Eindex^{ty\_2Equote\_2Eindex}) \tag{51}$$

Let $c\_2Equote\_2ENode\_\_vm : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Equote\_2ENode\_\_vm\ A\_27a \in ((((ty\_2Equote\_2Evarmap\ A\_27a)^{(ty\_2Equote\_2Evarmap\ A\_27a)})^{(ty\_2Equote\_2Evarmap\ A\_27a)})^{A\_27a}) \tag{52}$$

Let $c\_2Equote\_2EEnd\_\_idx : \iota$ be given. Assume the following.

$$c\_2Equote\_2EEnd\_\_idx \in ty\_2Equote\_2Eindex \tag{53}$$

Let $c\_2Equote\_2Evarmap\_\_find : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Equote\_2Evarmap\_\_find\ A\_27a \in ((A\_27a^{(ty\_2Equote\_2Evarmap\ A\_27a)})^{ty\_2Equote\_2Eindex}) \tag{54}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{55}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0t2 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum$$
$$A\_27a).(\forall V1t1 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).$$
$$(\forall V2sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V3l2 \in$$
$$(ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V4l1 \in (ty\_2Elist\_2Elist$$
$$ty\_2Equote\_2Eindex).(\forall V5c2 \in A\_27a.(\forall V6c1 \in A\_27a.$$
$$((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge\ A\_27a)\ V2sr)$$
$$(ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V6c1)\ V4l1)\ V1t1))$$
$$(ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V5c2)\ V3l2)\ V0t2)) =$$
$$(ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2EternaryComparisons\_2Eordering\_\_CASE}\ (ty\_2Ecanonical\_2Ecanonical\_\_sum$$
$$A\_27a))\ (ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_\_compare$$
$$ty\_2Equote\_2Eindex\ ty\_2Equote\_2Eindex)\ c\_2Equote\_2Eindex\_\_compare)$$
$$V4l1)\ V3l2))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V6c1)$$
$$V4l1)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge\ A\_27a)$$
$$V2sr)\ V1t1)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V5c2)$$
$$V3l2)\ V0t2))))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)$$
$$(ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a)\ V2sr)\ V6c1)$$
$$V5c2))\ V4l1)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge$$
$$A\_27a)\ V2sr)\ V1t1)\ V0t2)))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom$$
$$A\_27a)\ V5c2)\ V3l2)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge$$
$$A\_27a)\ V2sr)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V6c1)$$
$$V4l1)\ V1t1))\ V0t2))))))))))) \wedge ((\forall V7t2 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum$$
$$A\_27a).(\forall V8t1 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).$$
$$(\forall V9sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V10l2 \in$$
$$(ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V11l1 \in (ty\_2Elist\_2Elist$$
$$ty\_2Equote\_2Eindex).(\forall V12c1 \in A\_27a.((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge$$
$$A\_27a)\ V9sr)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V12c1)$$
$$V11l1)\ V8t1))\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V10l2)$$
$$V7t2)) = (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2EternaryComparisons\_2Eordering\_\_CASE}$$
$$(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a))\ (ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_\_compare$$
$$ty\_2Equote\_2Eindex\ ty\_2Equote\_2Eindex)\ c\_2Equote\_2Eindex\_\_compare)$$
$$V11l1)\ V10l2))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)$$
$$V12c1)\ V11l1)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge$$
$$A\_27a)\ V9sr)\ V8t1)\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)$$
$$V10l2)\ V7t2))))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)$$
$$(ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a)\ V9sr)\ V12c1)$$
$$(ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1\ A\_27a)\ V9sr)))\ V11l1)$$
$$(ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge\ A\_27a)\ V9sr)$$
$$V8t1)\ V7t2)))\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V10l2)$$
$$(ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge\ A\_27a)\ V9sr)$$
$$(ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V12c1)\ V11l1)$$
$$V8t1))\ V7t2)))))))))) \wedge ((\forall V13t2 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum$$
$$A\_27a).(\forall V14t1 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).$$
$$(\forall V15sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V16l2 \in$$
$$(ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V17l1 \in (ty\_2Elist\_2Elist$$
$$ty\_2Equote\_2Eindex).(\forall V18c2 \in A\_27a.((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge$$
$$A\_27a)\ V15sr)\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V17l1)$$
$$V14t1))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V18c2)$$
$$V16l2)\ V13t2)) = (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2EternaryComparisons\_2Eordering\_\_CASE}$$
$$(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a))\ (ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_\_compare$$
$$ty\_2Equote\_2Eindex\ ty\_2Equote\_2Eindex)\ c\_2Equote\_2Eindex\_\_compare)$$
$$V17l1)\ V16l2))\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)$$
$$V17l1)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge\ A\_27a)$$
$$V15sr)\ V14t1)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)$$
$$V18c2)\ V16l2)\ V13t2))))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom$$
$$A\_27a)\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a)\ V15sr)$$
$$(ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1\ A\_27a)\ V15sr))\ V18c2))$$
$$V17l1)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge\ A\_27a)$$
$$V15sr)\ V14t1)\ V13t2)))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom$$
$$A\_27a)\ V18c2)\ V16l2)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge$$

Assume the following.

$$
\forall A\_27a.\mathit{nonempty}\ A\_27a \Rightarrow ((\forall V0t2 \in (\mathit{ty\_2Ecanonical\_2Ecanonical\_\_sum}\ A\_27a).(\forall V1sr \in (\mathit{ty\_2Esemi\_\_ring\_2Esemi\_\_ring}\ A\_27a).
(\forall V2l2 \in (\mathit{ty\_2Elist\_2Elist}\ \mathit{ty\_2Equote\_2Eindex}).(\forall V3l1 \in (\mathit{ty\_2Elist\_2Elist}\ \mathit{ty\_2Equote\_2Eindex}).(\forall V4c2 \in A\_27a.
(\forall V5c1 \in A\_27a.((ap\ (ap\ (ap\ (ap\ (\mathit{c\_2Ecanonical\_2Emonom\_\_insert}\ A\_27a)\ V1sr)\ V5c1)\ V3l1)\ (ap\ (ap\ (ap\ (\mathit{c\_2Ecanonical\_2ECons\_\_monom}\ A\_27a)\ V4c2)\ V2l2)\ V0t2)) = (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2EternaryComparisons\_2Eordering\_\_CASE}\ (\mathit{ty\_2Ecanonical\_2Ecanonical\_\_sum}\ A\_27a))\ (ap\ (ap\ (ap\ (\mathit{c\_2EternaryComparisons\_2Elist\_\_compare}\ \mathit{ty\_2Equote\_2Eindex}\ \mathit{ty\_2Equote\_2Eindex})\ \mathit{c\_2Equote\_2Eindex\_\_compare})\ V3l1)\ V2l2))\ (ap\ (ap\ (ap\ (\mathit{c\_2Ecanonical\_2ECons\_\_monom}\ A\_27a)\ V5c1)\ V3l1)\ (ap\ (ap\ (ap\ (\mathit{c\_2Ecanonical\_2ECons\_\_monom}\ A\_27a)\ V4c2)\ V2l2)\ V0t2)))\ (ap\ (ap\ (ap\ (\mathit{c\_2Ecanonical\_2ECons\_\_monom}\ A\_27a)\ (ap\ (ap\ (ap\ (\mathit{c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP}\ A\_27a)\ V1sr)\ V5c1)\ V4c2))\ V3l1)\ V0t2))\ (ap\ (ap\ (ap\ (\mathit{c\_2Ecanonical\_2ECons\_\_monom}\ A\_27a)\ V4c2)\ V2l2)\ (ap\ (ap\ (ap\ (ap\ (\mathit{c\_2Ecanonical\_2Emonom\_\_insert}\ A\_27a)\ V1sr)\ V5c1)\ V3l1)\ V0t2))))))))))) \wedge ((\forall V6t2 \in (\mathit{ty\_2Ecanonical\_2Ecanonical\_\_sum}\ A\_27a).(\forall V7sr \in (\mathit{ty\_2Esemi\_\_ring\_2Esemi\_\_ring}\ A\_27a).
(\forall V8l2 \in (\mathit{ty\_2Elist\_2Elist}\ \mathit{ty\_2Equote\_2Eindex}).(\forall V9l1 \in (\mathit{ty\_2Elist\_2Elist}\ \mathit{ty\_2Equote\_2Eindex}).(\forall V10c1 \in A\_27a.
((ap\ (ap\ (ap\ (ap\ (\mathit{c\_2Ecanonical\_2Emonom\_\_insert}\ A\_27a)\ V7sr)\ V10c1)\ V9l1)\ (ap\ (ap\ (\mathit{c\_2Ecanonical\_2ECons\_\_varlist}\ A\_27a)\ V8l2)\ V6t2)) = (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2EternaryComparisons\_2Eordering\_\_CASE}\ (\mathit{ty\_2Ecanonical\_2Ecanonical\_\_sum}\ A\_27a))\ (ap\ (ap\ (ap\ (\mathit{c\_2EternaryComparisons\_2Elist\_\_compare}\ \mathit{ty\_2Equote\_2Eindex}\ \mathit{ty\_2Equote\_2Eindex})\ \mathit{c\_2Equote\_2Eindex\_\_compare})\ V9l1)\ V8l2))\ (ap\ (ap\ (ap\ (\mathit{c\_2Ecanonical\_2ECons\_\_monom}\ A\_27a)\ V10c1)\ V9l1)\ (ap\ (ap\ (\mathit{c\_2Ecanonical\_2ECons\_\_varlist}\ A\_27a)\ V8l2)\ V6t2)))\ (ap\ (ap\ (ap\ (\mathit{c\_2Ecanonical\_2ECons\_\_monom}\ A\_27a)\ (ap\ (ap\ (ap\ (\mathit{c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP}\ A\_27a)\ V7sr)\ V10c1)\ (ap\ (\mathit{c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1}\ A\_27a)\ V7sr)))\ V9l1)\ V6t2))\ (ap\ (ap\ (\mathit{c\_2Ecanonical\_2ECons\_\_varlist}\ A\_27a)\ V8l2)\ (ap\ (ap\ (ap\ (ap\ (\mathit{c\_2Ecanonical\_2Emonom\_\_insert}\ A\_27a)\ V7sr)\ V10c1)\ V9l1)\ V6t2))))))))) \wedge (\forall V11sr \in (\mathit{ty\_2Esemi\_\_ring\_2Esemi\_\_ring}\ A\_27a).(\forall V12l1 \in (\mathit{ty\_2Elist\_2Elist}\ \mathit{ty\_2Equote\_2Eindex}).
(\forall V13c1 \in A\_27a.((ap\ (ap\ (ap\ (ap\ (\mathit{c\_2Ecanonical\_2Emonom\_\_insert}\ A\_27a)\ V11sr)\ V13c1)\ V12l1)\ (\mathit{c\_2Ecanonical\_2ENil\_\_monom}\ A\_27a)) = (ap\ (ap\ (ap\ (\mathit{c\_2Ecanonical\_2ECons\_\_monom}\ A\_27a)\ V13c1)\ V12l1)\ (\mathit{c\_2Ecanonical\_2ENil\_\_monom}\ A\_27a))))))))
\tag{57}
$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0t2 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).(\forall V1sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V2l2 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V3l1 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V4c2 \in A\_27a.((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Evarlist\_\_insert\ A\_27a)\ V1sr)\ V3l1)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V4c2)\ V2l2)\ V0t2)) = (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2EternaryComparisons\_2Eordering\_\_CASE}\ (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a))\ (ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_\_compare\ ty\_2Equote\_2Eindex\ ty\_2Equote\_2Eindex)\ c\_2Equote\_2Eindex\_\_compare)\ V3l1)\ V2l2))\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V3l1)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V4c2)\ V2l2)\ V0t2)))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a)\ V1sr)\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1\ A\_27a)\ V1sr))\ V4c2))\ V3l1)\ V0t2))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V4c2)\ V2l2)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Evarlist\_\_insert\ A\_27a)\ V1sr)\ V3l1)\ V0t2)))))))))) \wedge ((\forall V5t2 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).(\forall V6sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V7l2 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V8l1 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Evarlist\_\_insert\ A\_27a)\ V6sr)\ V8l1)\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V7l2)\ V5t2)) = (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2EternaryComparisons\_2Eordering\_\_CASE}\ (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a))\ (ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_\_compare\ ty\_2Equote\_2Eindex\ ty\_2Equote\_2Eindex)\ c\_2Equote\_2Eindex\_\_compare)\ V8l1)\ V7l2))\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V8l1)\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V7l2)\ V5t2)))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a)\ V6sr)\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1\ A\_27a)\ V6sr))\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1\ A\_27a)\ V6sr)))\ V8l1)\ V5t2))\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V7l2)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Evarlist\_\_insert\ A\_27a)\ V6sr)\ V8l1)\ V5t2)))))))) \wedge (\forall V9sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V10l1 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Evarlist\_\_insert\ A\_27a)\ V9sr)\ V10l1)\ (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a)) = (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V10l1)\ (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a))))))) \quad (58)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V1c0 \in A\_27a.(\forall V2c \in A\_27a.(\forall V3l \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V4t \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar\ A\_27a)\ V0sr)\ V1c0)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V2c)\ V3l)\ V4t)) = (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM\ A\_27a)\ V0sr)\ V1c0)\ V2c))\ V3l)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar\ A\_27a)\ V0sr)\ V1c0)\ V4t)))))))) \wedge ((\forall V5sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V6c0 \in A\_27a.(\forall V7l \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V8t \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar\ A\_27a)\ V5sr)\ V6c0)\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V7l)\ V8t)) = (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V6c0)\ V7l)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar\ A\_27a)\ V5sr)\ V6c0)\ V8t))))))) \wedge (\forall V9sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V10c0 \in A\_27a.((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar\ A\_27a)\ V9sr)\ V10c0)\ (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a)) = (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a)))))) \tag{59}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V1l0 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V2c \in A\_27a.(\forall V3l \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V4t \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar2\ A\_27a)\ V0sr)\ V1l0)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V2c)\ V3l)\ V4t)) = (ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Emonom\_\_insert\ A\_27a)\ V0sr)\ V2c)\ (ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_\_merge\ ty\_2Equote\_2Eindex)\ \mathsf{c\_2Equote\_2Eindex\_\_lt})\ V1l0)\ V3l))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar2\ A\_27a)\ V0sr)\ V1l0)\ V4t)))))))) \wedge ((\forall V5sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V6l0 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V7l \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V8t \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar2\ A\_27a)\ V5sr)\ V6l0)\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V7l)\ V8t)) = (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Evarlist\_\_insert\ A\_27a)\ V5sr)\ (ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_\_merge\ ty\_2Equote\_2Eindex)\ \mathsf{c\_2Equote\_2Eindex\_\_lt})\ V6l0)\ V7l))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar2\ A\_27a)\ V5sr)\ V6l0)\ V8t))))))) \wedge (\forall V9sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V10l0 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar2\ A\_27a)\ V9sr)\ V10l0)\ (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a)) = (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a)))))) \tag{60}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V1c0 \in A\_27a.(\forall V2l0 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V3c \in A\_27a.(\forall V4l \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V5t \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar3\ A\_27a)\ V0sr)\ V1c0)\ V2l0)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V3c)\ V4l)\ V5t)) = (ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Emonom\_\_insert\ A\_27a)\ V0sr)\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM\ A\_27a)\ V0sr)\ V1c0)\ V3c))\ (ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_\_merge\ ty\_2Equote\_2Eindex)\ \mathsf{c\_2Equote\_2Eindex\_\_lt})\ V2l0)\ V4l))\ (ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar3\ A\_27a)\ V0sr)\ V1c0)\ V2l0)\ V5t)))))))))\wedge((\forall V6sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V7c0 \in A\_27a.(\forall V8l0 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V9l \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V10t \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar3\ A\_27a)\ V6sr)\ V7c0)\ V8l0)\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V9l)\ V10t)) = (ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Emonom\_\_insert\ A\_27a)\ V6sr)\ V7c0)\ (ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_\_merge\ ty\_2Equote\_2Eindex)\ \mathsf{c\_2Equote\_2Eindex\_\_lt})\ V8l0)\ V9l))\ (ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar3\ A\_27a)\ V6sr)\ V7c0)\ V8l0)\ V10t))))))))\wedge(\forall V11sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V12c0 \in A\_27a.(\forall V13l0 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).((ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar3\ A\_27a)\ V11sr)\ V12c0)\ V13l0)\ (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a)) = (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a))))))) \tag{61}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V1c1 \in A\_27a.(\forall V2l1 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V3t1 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).(\forall V4s2 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_prod\ A\_27a)\ V0sr)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V1c1)\ V2l1)\ V3t1))\ V4s2) = (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge\ A\_27a)\ V0sr)\ (ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar3\ A\_27a)\ V0sr)\ V1c1)\ V2l1)\ V4s2))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_prod\ A\_27a)\ V0sr)\ V3t1)\ V4s2)))))))) \wedge ((\forall V5sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V6l1 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V7t1 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).(\forall V8s2 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_prod\ A\_27a)\ V5sr)\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V6l1)\ V7t1))\ V8s2) = (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge\ A\_27a)\ V5sr)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar2\ A\_27a)\ V5sr)\ V6l1)\ V8s2))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_prod\ A\_27a)\ V5sr)\ V7t1)\ V8s2))))))) \wedge (\forall V9sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V10s2 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_prod\ A\_27a)\ V9sr)\ (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a))\ V10s2) = (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a)))))) \quad (62)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V1c \in A\_27a.(\forall V2l \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V3t \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_simplify\ A\_27a)\ V0sr)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V1c)\ V2l)\ V3t)) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ A\_27a)\ V1c)\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR0\ A\_27a)\ V0sr)))\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_simplify\ A\_27a)\ V0sr)\ V3t))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ A\_27a)\ V1c)\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1\ A\_27a)\ V0sr)))\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V2l)\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_simplify\ A\_27a)\ V0sr)\ V3t)))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V1c)\ V2l)\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_simplify\ A\_27a)\ V0sr)\ V3t)))))))))) \wedge ((\forall V4sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V5l \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V6t \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_simplify\ A\_27a)\ V4sr)\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V5l)\ V6t)) = (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V5l)\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_simplify\ A\_27a)\ V4sr)\ V6t)))))) \wedge (\forall V7sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).((ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_simplify\ A\_27a)\ V7sr)\ (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a)) = (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a))))) \tag{63}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V1vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V2x \in ty\_2Equote\_2Eindex.((ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Eivl\_\_aux\ A\_27a)\ V0sr)\ V1vm)\ V2x)\ (c\_2Elist\_2ENIL\ ty\_2Equote\_2Eindex)) = (ap\ (ap\ (c\_2Equote\_2Evarmap\_\_find\ A\_27a)\ V2x)\ V1vm))))) \wedge (\forall V3sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V4vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V5x \in ty\_2Equote\_2Eindex.(\forall V6x\_27 \in ty\_2Equote\_2Eindex.(\forall V7t\_27 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).((ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Eivl\_\_aux\ A\_27a)\ V3sr)\ V4vm)\ V5x)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Equote\_2Eindex)\ V6x\_27)\ V7t\_27)) = (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM\ A\_27a)\ V3sr)\ (ap\ (ap\ (c\_2Equote\_2Evarmap\_\_find\ A\_27a)\ V5x)\ V4vm))\ (ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Eivl\_\_aux\ A\_27a)\ V3sr)\ V4vm)\ V6x\_27)\ V7t\_27))))))))) \tag{64}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V1vm \in (ty\_2Equote\_2Evarmap\ A\_27a).((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_vl\ A\_27a)\ V0sr)\ V1vm)\ (c\_2Elist\_2ENIL\ ty\_2Equote\_2Eindex)) = (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1\ A\_27a)\ V0sr)))) \wedge (\forall V2sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V3vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V4x \in ty\_2Equote\_2Eindex.(\forall V5t \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_vl\ A\_27a)\ V2sr)\ V3vm)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Equote\_2Eindex)\ V4x)\ V5t)) = (ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Eivl\_\_aux\ A\_27a)\ V2sr)\ V3vm)\ V4x)\ V5t))))))) \tag{65}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V1vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V2c \in A\_27a.((ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_m\ A\_27a)\ V0sr)\ V1vm)\ V2c)\ (c\_2Elist\_2ENIL\ ty\_2Equote\_2Eindex)) = V2c)))) \wedge (\forall V3sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V4vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V5c \in A\_27a.(\forall V6x \in ty\_2Equote\_2Eindex.(\forall V7t \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).((ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_m\ A\_27a)\ V3sr)\ V4vm)\ V5c)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Equote\_2Eindex)\ V6x)\ V7t)) = (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM\ A\_27a)\ V3sr)\ V5c)\ (ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Eivl\_\_aux\ A\_27a)\ V3sr)\ V4vm)\ V6x)\ V7t))))))))) \tag{66}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V1vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V2a \in A\_27a.((ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Eics\_\_aux\ A\_27a)\ V0sr)\ V1vm)\ V2a)\ (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a)) = V2a)))) \wedge ((\forall V3sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V4vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V5a \in A\_27a.(\forall V6l \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V7t \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Eics\_\_aux\ A\_27a)\ V3sr)\ V4vm)\ V5a)\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V6l)\ V7t)) = (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a)\ V3sr)\ V5a)\ (ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Eics\_\_aux\ A\_27a)\ V3sr)\ V4vm)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_vl\ A\_27a)\ V3sr)\ V4vm)\ V6l))\ V7t)))))))) \wedge (\forall V8sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V9vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V10a \in A\_27a.(\forall V11c \in A\_27a.(\forall V12l \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V13t \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Eics\_\_aux\ A\_27a)\ V8sr)\ V9vm)\ V10a)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V11c)\ V12l)\ V13t)) = (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a)\ V8sr)\ V10a)\ (ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Eics\_\_aux\ A\_27a)\ V8sr)\ V9vm)\ (ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_m\ A\_27a)\ V8sr)\ V9vm)\ V11c)\ V12l))\ V13t)))))))))) \tag{67}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V1vm \in (ty\_2Equote\_2Evarmap\ A\_27a).((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_cs\ A\_27a)\ V0sr)\ V1vm)\ (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a)) = (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR0\ A\_27a)\ V0sr)))) \wedge ((\forall V2sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V3vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V4l \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V5t \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_cs\ A\_27a)\ V2sr)\ V3vm)\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V4l)\ V5t)) = (ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Eics\_\_aux\ A\_27a)\ V2sr)\ V3vm)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_vl\ A\_27a)\ V2sr)\ V3vm)\ V4l))\ V5t)))))) \wedge (\forall V6sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V7vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V8c \in A\_27a.(\forall V9l \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V10t \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_cs\ A\_27a)\ V6sr)\ V7vm)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V8c)\ V9l)\ V10t)) = (ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Eics\_\_aux\ A\_27a)\ V6sr)\ V7vm)\ (ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_m\ A\_27a)\ V6sr)\ V7vm)\ V8c)\ V9l))\ V10t))))))))) \tag{68}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V1i \in ty\_2Equote\_2Eindex.((ap\ (ap\ (c\_2Ecanonical\_2Espolynom\_\_normalize\ A\_27a)\ V0sr)\ (ap\ (c\_2Ecanonical\_2ESPvar\ A\_27a)\ V1i)) = (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Equote\_2Eindex)\ V1i)\ (c\_2Elist\_2ENIL\ ty\_2Equote\_2Eindex)))\ (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a))))) \wedge ((\forall V2sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V3c \in A\_27a.((ap\ (ap\ (c\_2Ecanonical\_2Espolynom\_\_normalize\ A\_27a)\ V2sr)\ (ap\ (c\_2Ecanonical\_2ESPconst\ A\_27a)\ V3c)) = (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V3c)\ (c\_2Elist\_2ENIL\ ty\_2Equote\_2Eindex))\ (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a))))) \wedge ((\forall V4sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V5l \in (ty\_2Ecanonical\_2Espolynom\ A\_27a).(\forall V6r \in (ty\_2Ecanonical\_2Espolynom\ A\_27a).((ap\ (ap\ (c\_2Ecanonical\_2Espolynom\_\_normalize\ A\_27a)\ V4sr)\ (ap\ (ap\ (c\_2Ecanonical\_2ESPplus\ A\_27a)\ V5l)\ V6r)) = (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge\ A\_27a)\ V4sr)\ (ap\ (ap\ (c\_2Ecanonical\_2Espolynom\_\_normalize\ A\_27a)\ V4sr)\ V5l))\ (ap\ (ap\ (c\_2Ecanonical\_2Espolynom\_\_normalize\ A\_27a)\ V4sr)\ V6r)))))) \wedge (\forall V7sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V8l \in (ty\_2Ecanonical\_2Espolynom\ A\_27a).(\forall V9r \in (ty\_2Ecanonical\_2Espolynom\ A\_27a).((ap\ (ap\ (c\_2Ecanonical\_2Espolynom\_\_normalize\ A\_27a)\ V7sr)\ (ap\ (ap\ (c\_2Ecanonical\_2ESPmult\ A\_27a)\ V8l)\ V9r)) = (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_prod\ A\_27a)\ V7sr)\ (ap\ (ap\ (c\_2Ecanonical\_2Espolynom\_\_normalize\ A\_27a)\ V7sr)\ V8l))\ (ap\ (ap\ (c\_2Ecanonical\_2Espolynom\_\_normalize\ A\_27a)\ V7sr)\ V9r))))))))) \tag{69}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V1vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V2c \in A\_27a.((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_sp\ A\_27a)\ V0sr)\ V1vm)\ (ap\ (c\_2Ecanonical\_2ESPconst\ A\_27a)\ V2c)) = V2c)))) \wedge ((\forall V3sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V4vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V5i \in ty\_2Equote\_2Eindex.((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_sp\ A\_27a)\ V3sr)\ V4vm)\ (ap\ (c\_2Ecanonical\_2ESPvar\ A\_27a)\ V5i)) = (ap\ (ap\ (c\_2Equote\_2Evarmap\_\_find\ A\_27a)\ V5i)\ V4vm))))) \wedge ((\forall V6sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V7vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V8p1 \in (ty\_2Ecanonical\_2Espolynom\ A\_27a).(\forall V9p2 \in (ty\_2Ecanonical\_2Espolynom\ A\_27a).((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_sp\ A\_27a)\ V6sr)\ V7vm)\ (ap\ (ap\ (c\_2Ecanonical\_2ESPplus\ A\_27a)\ V8p1)\ V9p2)) = (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a)\ V6sr)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_sp\ A\_27a)\ V6sr)\ V7vm)\ V8p1))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_sp\ A\_27a)\ V6sr)\ V7vm)\ V9p2))))))) \wedge (\forall V10sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V11vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V12p1 \in (ty\_2Ecanonical\_2Espolynom\ A\_27a).(\forall V13p2 \in (ty\_2Ecanonical\_2Espolynom\ A\_27a).((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_sp\ A\_27a)\ V10sr)\ V11vm)\ (ap\ (ap\ (c\_2Ecanonical\_2ESPmult\ A\_27a)\ V12p1)\ V13p2)) = (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM\ A\_27a)\ V10sr)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_sp\ A\_27a)\ V10sr)\ V11vm)\ V12p1))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_sp\ A\_27a)\ V10sr)\ V11vm)\ V13p2)))))))))) \tag{70}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).((p\ (ap\ (\mathsf{c\_2Esemi\_\_ring\_2Eis\_\_semi\_\_ring}\ A\_27a)\ V0sr)) \Rightarrow (\forall V1vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V2p \in (ty\_2Ecanonical\_2Espolynom\ A\_27a).((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_cs\ A\_27a)\ V0sr)\ V1vm)\ (ap\ (ap\ (\mathsf{c\_2Ecanonical\_2Espolynom\_\_simplify}\ A\_27a)\ V0sr)\ V2p)) = (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_sp\ A\_27a)\ V0sr)\ V1vm)\ V2p)))))) \tag{71}$$

Assume the following.

$$(p\ (ap\ (\mathsf{c\_2Esemi\_\_ring\_2Eis\_\_semi\_\_ring}\ ty\_2Enum\_2Enum)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ ty\_2Enum\_2Enum)\ \mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ c\_2Earithmetic\_2E\_2B)\ c\_2Earithmetic\_2E\_2A))) \tag{72}$$

Assume the following.

$$\begin{gathered}
\forall A_27a.\mathit{nonempty}\ A_27a \Rightarrow ((\forall V0x \in A_27a.(\forall V1v2 \in \\
(\mathit{ty_2Equote_2Evarmap}\ A_27a).(\forall V2v1 \in (\mathit{ty_2Equote_2Evarmap} \\
A_27a).((\mathit{ap}\ (\mathit{ap}\ (\mathit{c_2Equote_2Evarmap__find}\ A_27a)\ \mathit{c_2Equote_2EEnd__idx}) \\
(\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{c_2Equote_2ENode__vm}\ A_27a)\ V0x)\ V2v1)\ V1v2)) = V0x)))) \wedge \\
((\forall V3x \in A_27a.(\forall V4v2 \in (\mathit{ty_2Equote_2Evarmap}\ A_27a). \\
(\forall V5v1 \in (\mathit{ty_2Equote_2Evarmap}\ A_27a).(\forall V6i1 \in \mathit{ty_2Equote_2Eindex}. \\
((\mathit{ap}\ (\mathit{ap}\ (\mathit{c_2Equote_2Evarmap__find}\ A_27a)\ (\mathit{ap}\ \mathit{c_2Equote_2ERight__idx} \\
V6i1))\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{c_2Equote_2ENode__vm}\ A_27a)\ V3x)\ V5v1)\ V4v2)) = \\
(\mathit{ap}\ (\mathit{ap}\ (\mathit{c_2Equote_2Evarmap__find}\ A_27a)\ V6i1)\ V4v2)))))) \wedge ((\\
\forall V7x \in A_27a.(\forall V8v2 \in (\mathit{ty_2Equote_2Evarmap}\ A_27a). \\
(\forall V9v1 \in (\mathit{ty_2Equote_2Evarmap}\ A_27a).(\forall V10i1 \in \mathit{ty_2Equote_2Eindex}. \\
((\mathit{ap}\ (\mathit{ap}\ (\mathit{c_2Equote_2Evarmap__find}\ A_27a)\ (\mathit{ap}\ \mathit{c_2Equote_2ELeft__idx} \\
V10i1))\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{c_2Equote_2ENode__vm}\ A_27a)\ V7x)\ V9v1)\ V8v2)) = \\
(\mathit{ap}\ (\mathit{ap}\ (\mathit{c_2Equote_2Evarmap__find}\ A_27a)\ V10i1)\ V9v1)))))) \wedge (\\
\forall V11i \in \mathit{ty_2Equote_2Eindex}.((\mathit{ap}\ (\mathit{ap}\ (\mathit{c_2Equote_2Evarmap__find} \\
A_27a)\ V11i)\ (\mathit{c_2Equote_2EEmpty__vm}\ A_27a)) = (\mathit{ap}\ (\mathsf{c_2Emin_2E_40} \\
A_27a)\ (\lambda V12x \in A_27a.\mathsf{c_2Ebool_2ET})))))))
\end{gathered} \tag{73}$$

Assume the following.

$$\begin{gathered}
\forall A_27a.\mathit{nonempty}\ A_27a \Rightarrow ((\forall V0a \in A_27a.(\forall V1a0 \in \\
A_27a.(\forall V2f \in ((A_27a^{A_27a})^{A_27a}).(\forall V3f0 \in ((A_27a^{A_27a})^{A_27a}). \\
((\mathit{ap}\ (\mathit{c_2Esemi__ring_2Esemi__ring__SR0}\ A_27a)\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap} \\
(\mathit{c_2Esemi__ring_2Erecordtype_2Esemi__ring}\ A_27a)\ V0a)\ V1a0) \\
V2f)\ V3f0)) = V0a))))) \wedge ((\forall V4a \in A_27a.(\forall V5a0 \in A_27a. \\
(\forall V6f \in ((A_27a^{A_27a})^{A_27a}).(\forall V7f0 \in ((A_27a^{A_27a})^{A_27a}). \\
((\mathit{ap}\ (\mathit{c_2Esemi__ring_2Esemi__ring__SR1}\ A_27a)\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap} \\
(\mathit{c_2Esemi__ring_2Erecordtype_2Esemi__ring}\ A_27a)\ V4a)\ V5a0) \\
V6f)\ V7f0)) = V5a0))))) \wedge ((\forall V8a \in A_27a.(\forall V9a0 \in A_27a. \\
(\forall V10f \in ((A_27a^{A_27a})^{A_27a}).(\forall V11f0 \in ((A_27a^{A_27a})^{A_27a}). \\
((\mathit{ap}\ (\mathit{c_2Esemi__ring_2Esemi__ring__SRP}\ A_27a)\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap} \\
(\mathit{c_2Esemi__ring_2Erecordtype_2Esemi__ring}\ A_27a)\ V8a)\ V9a0) \\
V10f)\ V11f0)) = V10f))))) \wedge (\forall V12a \in A_27a.(\forall V13a0 \in \\
A_27a.(\forall V14f \in ((A_27a^{A_27a})^{A_27a}).(\forall V15f0 \in ((\\
A_27a^{A_27a})^{A_27a}).((\mathit{ap}\ (\mathit{c_2Esemi__ring_2Esemi__ring__SRM} \\
A_27a)\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{c_2Esemi__ring_2Erecordtype_2Esemi__ring} \\
A_27a)\ V12a)\ V13a0)\ V14f)\ V15f0)) = V15f0))))))))
\end{gathered} \tag{74}$$

Theorem 1

$$
\begin{gathered}
\forall A\_27a.\mathit{nonempty}\ A\_27a \Rightarrow ((p\ (\mathit{ap}\ (\mathsf{c\_2Esemi\_\_ring\_2Eis\_\_semi\_\_ring} \\
\mathit{ty\_2Enum\_2Enum})\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring} \\
\mathit{ty\_2Enum\_2Enum})\ \mathsf{c\_2Enum\_2E0})\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\\
\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ \mathit{c\_2Earithmetic\_2E\_2B}) \\
\mathit{c\_2Earithmetic\_2E\_2A})))\wedge((\forall V0vm \in (\mathit{ty\_2Equote\_2Evarmap} \\
\mathit{ty\_2Enum\_2Enum}).(\forall V1p \in (\mathit{ty\_2Ecanonical\_2Espolynom}\ \mathit{ty\_2Enum\_2Enum}). \\
((\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2EnumRing\_2Enum\_\_interp\_\_sp}\ V0vm)\ V1p) = (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2EnumRing\_2Enum\_\_interp\_\_cs} \\
V0vm)\ (\mathit{ap}\ \mathsf{c\_2EnumRing\_2Enum\_\_spolynom\_\_simplify}\ V1p)))))\wedge(\\
(((\forall V2vm \in (\mathit{ty\_2Equote\_2Evarmap}\ \mathit{ty\_2Enum\_2Enum}).(\forall V3c \in \\
\mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2EnumRing\_2Enum\_\_interp\_\_sp}\ V2vm) \\
(\mathit{ap}\ (\mathit{c\_2Ecanonical\_2ESPconst}\ \mathit{ty\_2Enum\_2Enum})\ V3c)) = V3c)))\wedge \\
((\forall V4vm \in (\mathit{ty\_2Equote\_2Evarmap}\ \mathit{ty\_2Enum\_2Enum}).(\forall V5i \in \\
\mathit{ty\_2Equote\_2Eindex}.((\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2EnumRing\_2Enum\_\_interp\_\_sp} \\
V4vm)\ (\mathit{ap}\ (\mathit{c\_2Ecanonical\_2ESPvar}\ \mathit{ty\_2Enum\_2Enum})\ V5i)) = (\mathit{ap}\ (\\
\mathit{ap}\ (\mathit{c\_2Equote\_2Evarmap\_\_find}\ \mathit{ty\_2Enum\_2Enum})\ V5i)\ V4vm))))\wedge \\
((\forall V6vm \in (\mathit{ty\_2Equote\_2Evarmap}\ \mathit{ty\_2Enum\_2Enum}).(\forall V7p1 \in \\
(\mathit{ty\_2Ecanonical\_2Espolynom}\ \mathit{ty\_2Enum\_2Enum}).(\forall V8p2 \in \\
(\mathit{ty\_2Ecanonical\_2Espolynom}\ \mathit{ty\_2Enum\_2Enum}).((\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2EnumRing\_2Enum\_\_interp\_\_sp} \\
V6vm)\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{c\_2Ecanonical\_2ESPplus}\ \mathit{ty\_2Enum\_2Enum})\ V7p1)\ V8p2)) = \\
(\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2EnumRing\_2Enum\_\_interp\_\_sp} \\
V6vm)\ V7p1))\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2EnumRing\_2Enum\_\_interp\_\_sp}\ V6vm)\ V8p2))))))\wedge \\
(\forall V9vm \in (\mathit{ty\_2Equote\_2Evarmap}\ \mathit{ty\_2Enum\_2Enum}).(\forall V10p1 \in \\
(\mathit{ty\_2Ecanonical\_2Espolynom}\ \mathit{ty\_2Enum\_2Enum}).(\forall V11p2 \in \\
(\mathit{ty\_2Ecanonical\_2Espolynom}\ \mathit{ty\_2Enum\_2Enum}).((\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2EnumRing\_2Enum\_\_interp\_\_sp} \\
V9vm)\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{c\_2Ecanonical\_2ESPmult}\ \mathit{ty\_2Enum\_2Enum})\ V10p1) \\
V11p2)) = (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2A}\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2EnumRing\_2Enum\_\_interp\_\_sp} \\
V9vm)\ V10p1))\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2EnumRing\_2Enum\_\_interp\_\_sp}\ V9vm)\ V11p2)))))))))\wedge \\
((\forall V12x \in A\_27a.(\forall V13v2 \in (\mathit{ty\_2Equote\_2Evarmap}\ A\_27a). \\
(\forall V14v1 \in (\mathit{ty\_2Equote\_2Evarmap}\ A\_27a).((\mathit{ap}\ (\mathit{ap}\ (\mathit{c\_2Equote\_2Evarmap\_\_find} \\
A\_27a)\ \mathit{c\_2Equote\_2EEnd\_\_idx})\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{c\_2Equote\_2ENode\_\_vm} \\
A\_27a)\ V12x)\ V14v1)\ V13v2)) = V12x))))\wedge((\forall V15x \in A\_27a.(\\
\forall V16v2 \in (\mathit{ty\_2Equote\_2Evarmap}\ A\_27a).(\forall V17v1 \in (\\
\mathit{ty\_2Equote\_2Evarmap}\ A\_27a).(\forall V18i1 \in \mathit{ty\_2Equote\_2Eindex}. \\
((\mathit{ap}\ (\mathit{ap}\ (\mathit{c\_2Equote\_2Evarmap\_\_find}\ A\_27a)\ (\mathit{ap}\ \mathit{c\_2Equote\_2ERight\_\_idx} \\
V18i1))\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{c\_2Equote\_2ENode\_\_vm}\ A\_27a)\ V15x)\ V17v1)\ V16v2)) = \\
(\mathit{ap}\ (\mathit{ap}\ (\mathit{c\_2Equote\_2Evarmap\_\_find}\ A\_27a)\ V18i1)\ V16v2))))))\wedge \\
((\forall V19x \in A\_27a.(\forall V20v2 \in (\mathit{ty\_2Equote\_2Evarmap}\ A\_27a). \\
(\forall V21v1 \in (\mathit{ty\_2Equote\_2Evarmap}\ A\_27a).(\forall V22i1 \in \\
\mathit{ty\_2Equote\_2Eindex}.((\mathit{ap}\ (\mathit{ap}\ (\mathit{c\_2Equote\_2Evarmap\_\_find}\ A\_27a) \\
(\mathit{ap}\ \mathit{c\_2Equote\_2ELeft\_\_idx}\ V22i1))\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{c\_2Equote\_2ENode\_\_vm} \\
A\_27a)\ V19x)\ V21v1)\ V20v2)) = (\mathit{ap}\ (\mathit{ap}\ (\mathit{c\_2Equote\_2Evarmap\_\_find} \\
A\_27a)\ V22i1)\ V21v1))))))\wedge(\forall V23i \in \mathit{ty\_2Equote\_2Eindex}. \\
((\mathit{ap}\ (\mathit{ap}\ (\mathit{c\_2Equote\_2Evarmap\_\_find}\ A\_27a)\ V23i)\ (\mathit{c\_2Equote\_2EEmpty\_\_vm} \\
A\_27a)) = (\mathit{ap}\ (\mathsf{c\_2Emin\_2E\_40}\ A\_27a)\ (\lambda V24x \in A\_27a.\mathsf{c\_2Ebool\_2ET}))))))))\wedge \\
(((\forall V25t2 \in (\mathit{ty\_2Ecanonical\_2Ecanonical\_\_sum}\ \mathit{ty\_2Enum\_2Enum}). \\
(\forall V26t1 \in (\mathit{ty\_2Ecanonical\_2Ecanonical\_\_sum}\ \mathit{ty\_2Enum\_2Enum}). \\
(\forall V27l2 \in (\mathit{ty\_2Elist\_2Elist}\ \mathit{ty\_2Equote\_2Eindex}).(\forall V28l1 \in \\
(\mathit{ty\_2Elist\_2Elist}\ \mathit{ty\_2Equote\_2Eindex}).(\forall V29c2 \in \mathit{ty\_2Enum\_2Enum}. \\
(\forall V30c1 \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2EnumRing\_2Enum\_\_canonical\_\_sum\_\_merge} \\
(\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{c\_2Ecanonical\_2ECons\_\_monom}\ \mathit{ty\_2Enum\_2Enum})\ V30c1) \\
V28l1)\ V26t1))\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{c\_2Ecanonical\_2ECons\_\_monom}\ \mathit{ty\_2Enum\_2Enum}) \\
V29c2)\ V27l2)\ V25t2)) = (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2EternaryComparisons\_2Eordering\_\_CASE} \\
(\mathit{ty\_2Ecanonical\_2Ecanonical\_\_sum}\ \mathit{ty\_2Enum\_2Enum}))\ (\mathit{ap}\ (\mathit{ap}\ (\\
\mathit{ap}\ (\mathit{c\_2EternaryComparisons\_2Elist\_\_compare}\ \mathit{ty\_2Equote\_2Eindex} \\
\mathit{ty\_2Equote\_2Eindex})\ \mathit{c\_2Equote\_2Eindex\_\_compare})\ V28l1)\ V27l2)) \\
(\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{c\_2Ecanonical\_2ECons\_\_monom}\ \mathit{ty\_2Enum\_2Enum})\ V30c1) \\
V28l1)\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2EnumRing\_2Enum\_\_canonical\_\_sum\_\_merge}\ V26t1) \\
(\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{c\_2Ecanonical\_2ECons\_\_monom}\ \mathit{ty\_2Enum\_2Enum})\ V29c2) \\
V27l2)\ V25t2))))\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{c\_2Ecanonical\_2ECons\_\_monom}\ \mathit{ty\_2Enum\_2Enum})
\end{gathered}
$$