

thm_2Enumeral_2ETWO__EXP__THM
(TMK8AwtLNWbhNfzuRiticHrGbkwf6FfwLTS)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{2}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 3 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{5}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{6}$$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 5 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (c_2Enum_2EREP_num\ c_2Enum_2ESUC_REP))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Definition 6 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B V0n) c_2Enum_2E0)$.

Definition 7 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Definition 8 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B V0n) c_2Enum_2E0)$.

Definition 9 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (8)$$

Let $c_2Enumeral_2Etepx_help : \iota$ be given. Assume the following.

$$c_2Enumeral_2Etepx_help \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (9)$$

Definition 10 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 11 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 12 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 13 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$.

Definition 14 We define c_2Ebool_2ECOND to be $\lambda A.\lambda a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.\lambda V2t2 \in A.\lambda V2a.((ap (ap c_2Ebool_2E_21 2) (ap V2a V2t2)))))$.

Definition 15 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap (ap (ap (c_2Ebool_2E_21 2) (ap V0m V0m)))))$.

Assume the following.

$$\begin{aligned} & ((\forall V0m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\ & V0m) c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\ & c_2Earithmetic_2EZERO)))) \wedge (\forall V1m \in ty_2Enum_2Enum.(\forall V2n \in \\ & ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V1m) (ap c_2Enum_2ESUC \\ & V2n)) = (ap (ap c_2Earithmetic_2E_2A V1m) (ap (ap c_2Earithmetic_2EEXP \\ & V1m) V2n)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} & ((\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\ & c_2Enum_2E0) V0m) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2B \\ & c_2Enum_2E0) V0m) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2B \\ & (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B \\ & V0m) V1n))) \wedge (((ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Enum_2ESUC \\ & V1n)) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n)))))))))) \end{aligned} \quad (11)$$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (13)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Enumeral_2Etexp_help \\ & V0n) c_2Enum_2E0) = (ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) (ap (ap c_2Earithmetic_2E_2B \\ & V0n) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\ & c_2Earithmetic_2EZERO)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & (((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = c_2Enum_2E0) \wedge (\forall V0m \in \\ & ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE (ap c_2Enum_2ESUC V0m)) = \\ & V0m))) \end{aligned} \quad (15)$$

Theorem 1

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum.(((ap (ap c_2Earithmetic_2EEXP \\ & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) \\ & c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\ & c_2Earithmetic_2EZERO))) \wedge (((ap (ap c_2Earithmetic_2EEXP (ap \\ & c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) \\ & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V0n))) = \\ & (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Enumeral_2Etexp_help \\ & (ap c_2Eprim_rec_2EPRE (ap c_2Earithmetic_2EBIT1 V0n))) c_2Earithmetic_2EZERO))) \wedge \\ & ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL (\\ & ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT2 V0n))) = (ap c_2Earithmetic_2ENUMERAL \\ & (ap (ap c_2Enumeral_2Etexp_help (ap c_2Earithmetic_2EBIT1 V0n)) \\ & c_2Earithmetic_2EZERO)))))) \end{aligned}$$