

thm\_2Enumeral\_2Edivmod\_\_POS  
(TMXdvzTJSZfnrfuQqBASyJyTogJD2s1ngeV)

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Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 3** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{5}$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x)))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ (ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))))$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

**Definition 7** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2EBIT1))$

**Definition 8** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 9** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2EBIT2))$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (7)$$

**Definition 10** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 11** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21) 2)) (\lambda V2t \in 2.V0t))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (8)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (9)$$

**Definition 12** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epair\_2E\_2C))$

**Definition 13** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21) 2) (\lambda V0t \in 2.V0t)$ .

**Definition 14** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_7E))$

**Definition 15** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p x)$ ) of type  $\iota \Rightarrow \iota$ .

**Definition 16** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40))))$

**Definition 17** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.V0m$

**Definition 18** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.V0t)))$

**Definition 19** We define  $c\_2Ebool\_2ELET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27b.V0f x))$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2ESND A\_27a A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod A\_27a A\_27b)}) \quad (10)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EFST A\_27a A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod A\_27a A\_27b)}) \quad (11)$$

**Definition 20** We define  $c\_Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c)^{A\_27a})$

Let  $c\_Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (12)$$

**Definition 21** We define  $c\_ERelation\_2Einv\_image$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27b})^{A\_27a}).\lambda V1$

**Definition 22** We define  $c\_Eprim\_rec\_2Emeasure$  to be  $\lambda A\_27a : \iota.(ap (c\_ERelation\_2Einv\_image A\_27a ty\_2Enum\_2Enum))$

Let  $c\_Ebool\_2EARB : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_Ebool\_2EARB A\_27a \in A\_27a \quad (13)$$

**Definition 23** We define  $c\_ERelation\_2ERESTRICT$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b)^{A\_27a}.\lambda V1$

**Definition 24** We define  $c\_ERelation\_2ETC$  to be  $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b$

**Definition 25** We define  $c\_ERelation\_2Eapprox$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

**Definition 26** We define  $c\_ERelation\_2Ethe\_fun$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

**Definition 27** We define  $c\_ERelation\_2EWFREC$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

**Definition 28** We define  $c\_Earithmetic\_2Efindq$  to be  $(ap (ap (c\_ERelation\_2EWFREC (ty\_2Epair\_2Eprod))))$

**Definition 29** We define  $c\_Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b)^{A\_27c}.\lambda V1$

**Definition 30** We define  $c\_Earithmetic\_2EDIVMOD$  to be  $(ap (ap (c\_ERelation\_2EWFREC (ty\_2Epair\_2Eprod))))$

**Definition 31** We define  $c\_Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_2E\_21) 2)) (\lambda V2t \in 2))$

**Definition 32** We define  $c\_Ebool\_2EBOUNDED$  to be  $(\lambda V0v \in 2.c\_Ebool\_2ET)$ .

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& \quad \forall V2n \in ty\_2Enum\_2Enum. ((ap\ c\_2Earithmetic\_2EDIVMOD\ (ap \\
& \quad (ap\ (c\_2Epair\_2E\_2C\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum \\
& \quad ty\_2Enum\_2Enum))\ V0a)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ ty\_2Enum\_2Enum \\
& \quad ty\_2Enum\_2Enum)\ V1m)\ V2n))) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ (ty\_2Epair\_2Eprod \\
& \quad ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))\ (ap\ (ap\ (c\_2Emin\_2E\_3D\ ty\_2Enum\_2Enum) \\
& \quad V2n)\ c\_2Enum\_2E0))\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum) \\
& \quad c\_2Enum\_2E0)\ c\_2Enum\_2E0))\ (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ (ty\_2Epair\_2Eprod \\
& \quad ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C \\
& \quad V1m)\ V2n))\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum) \\
& \quad V0a)\ V1m))\ (ap\ (ap\ (c\_2Ebool\_2ELET\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod \\
& \quad ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))\ (\lambda V3q \in ty\_2Enum\_2Enum. ( \\
& \quad ap\ c\_2Earithmetic\_2EDIVMOD\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ ty\_2Enum\_2Enum \\
& \quad (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& \quad V0a)\ V3q))\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum) \\
& \quad (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V1m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A \\
& \quad V2n)\ V3q)))\ V2n))))))\ (ap\ c\_2Earithmetic\_2Efindq\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\
& \quad ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)) \\
& \quad (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))) \\
& \quad (ap\ (ap\ (c\_2Epair\_2E\_2C\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ V1m)\ V2n)))))))))) \\
& \hspace{15em} (14)
\end{aligned}$$

Assume the following.

$$True \hspace{15em} (15)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\
& \quad V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \hspace{2em} (16)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \hspace{15em} (17)$$

Assume the following.

$$(\forall V0t \in 2. ((p\ V0t) \vee (\neg(p\ V0t)))) \hspace{15em} (18)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\
& (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \hspace{2em} (19)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (( \\
& (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \hspace{2em} (20)
\end{aligned}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (21)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (23)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in A\_27a. (((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF)\ V0t1)\ V1t2) = V1t2)))))) \quad (24)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (25)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \quad (26)$$

Assume the following.

$$(\forall V0v \in 2. ((p\ (ap\ c\_2Ebool\_2EBOUNDED\ V0v)) \Leftrightarrow True)) \quad (27)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27b. (\forall V2a \in A\_27a. (\forall V3b \in A\_27b. (((ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \quad (28)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\neg(p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0n)\ V0n)))) \quad (29)$$

**Theorem 1**

$$\begin{aligned} & (\forall V0a \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\ & \forall V2n \in ty\_2Enum\_2Enum. ((p (ap (ap (c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) \\ & V2n)) \Rightarrow ((ap c\_2Earithmetic\_2EDIVMOD (ap (ap (c\_2Epair\_2E\_2C ty\_2Enum\_2Enum \\ & (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)) V0a) (ap ( \\ & ap (c\_2Epair\_2E\_2C ty\_2Enum\_2Enum ty\_2Enum\_2Enum) V1m) V2n))) = \\ & (ap (ap (ap (c\_2Ebool\_2ECOND (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum \\ & ty\_2Enum\_2Enum)) (ap (ap c\_2Eprim\_rec\_2E\_3C V1m) V2n)) (ap (ap \\ & (c\_2Epair\_2E\_2C ty\_2Enum\_2Enum ty\_2Enum\_2Enum) V0a) V1m)) (ap \\ & (ap (c\_2Ebool\_2ELET ty\_2Enum\_2Enum (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum \\ & ty\_2Enum\_2Enum)) (\lambda V3q \in ty\_2Enum\_2Enum. (ap c\_2Earithmetic\_2EDIVMOD \\ & (ap (ap (c\_2Epair\_2E\_2C ty\_2Enum\_2Enum (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum \\ & ty\_2Enum\_2Enum)) (ap (ap c\_2Earithmetic\_2E\_2B V0a) V3q)) (ap ( \\ & ap (c\_2Epair\_2E\_2C ty\_2Enum\_2Enum ty\_2Enum\_2Enum) (ap (ap c\_2Earithmetic\_2E\_2D \\ & V1m) (ap (ap c\_2Earithmetic\_2E\_2A V2n) V3q))) V2n)))))) (ap c\_2Earithmetic\_2Efindq \\ & (ap (ap (c\_2Epair\_2E\_2C ty\_2Enum\_2Enum (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum \\ & ty\_2Enum\_2Enum)) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\ & c\_2Earithmetic\_2EZERO))) (ap (ap (c\_2Epair\_2E\_2C ty\_2Enum\_2Enum \\ & ty\_2Enum\_2Enum) V1m) V2n)))))))))) \end{aligned}$$