thm_2Enumeral_2Eenumeral__mult (TMWkkFuKJ3XqfTSETRxoCC34XffeFSsaidb)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 6 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 7 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A$

Definition 8 *We define* c_2Ecombin_2EC *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27c}$

Definition 9 *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g$

Definition 10 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_2}$

Definition 11 *We define* c_2Emarker_2EAbbrev *to be* $\lambda V0x \in 2.V0x$.

Definition 12 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 13 *We define* c_2Emarker_2EAC *to be* $\lambda V0b1 \in 2.\lambda V1b2 \in 2.(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ V0b1)$

Definition 14 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (2)$$

Definition 15 *We define* c_2Enumeral_2EiDUB *to be* $\lambda V0x \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2$

Definition 16 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \tag{3}$$

Definition 17 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enumeral\_2Etexp\_\_help : \iota$ be given. Assume the following.

$$c\_2Enumeral\_2Etexp\_\_help \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{5}$$

Let $c\_2Enumeral\_2Eonecount : \iota$ be given. Assume the following.

$$c\_2Enumeral\_2Eonecount \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

Definition 18 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 19 *We define* c_2Ebool_2ELET *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{7}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{8}$$

Definition 20 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 21 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{9}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{10}$$

Definition 22 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Definition 23 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 24 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{11}$$

Let $c\_2Enumeral\_2Eexactlog : \iota$ be given. Assume the following.

$$c\_2Enumeral\_2Eexactlog \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}) \tag{12}$$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{13}$$

Definition 25 *We define* c_2Enumeral_2Einternal__mult *to be* $c\_2Earithmetic\_2E\_2A$.

Definition 26 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{14}$$

Definition 27 *We define* c_2Earithmetic_2EDIV2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 28 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ ($c_2Ebool_2E

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((V0m = \mathsf{c\_2Enum\_2E0}) \lor (\exists V1n \in ty\_2Enum\_2Enum.(V0m = (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))))) \tag{15}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))) \tag{16}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \mathsf{c\_2Enum\_2E0})\ V0m) = \mathsf{c\_2Enum\_2E0}) \land (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0}) \land (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ V0m) = V0m) \land (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))) = V0m) \land (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n))\ V1n)) \land ((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n)))))))))) \tag{17}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V1n)\ V0m)))) \tag{18}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V1n)\ V2p)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n))\ V2p))))) \tag{19}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V2p))\Leftrightarrow((V0m = \mathsf{c\_2Enum\_2E0})\vee (V1n = V2p)))))) \tag{20}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V1n)\ V0m) = (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V2p)\ V0m))\Leftrightarrow((V0m = \mathsf{c\_2Enum\_2E0})\vee (V1n = V2p)))))) \tag{21}$$

Assume the following.

$$True \tag{22}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\ V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.(False\Rightarrow(p\ V0t))) \tag{24}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t)\vee(\neg(p\ V0t)))) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\forall V0f \in (A\_27b^{A\_27a}).(\forall V1x \in A\_27a.((ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ A\_27a\ A\_27b)\ V0f)\ V1x) = (ap\ V0f\ V1x)))) \tag{26}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{27}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{28}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{30}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{31}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2) = V1t2)))) \tag{33}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{34}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a.(\forall V5y\_27 \in A\_27a.((((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge ((\neg (p\ V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V1Q)\ V3x\_27)\ V5y\_27)))))))) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in A\_27a.(\exists V1x \in A\_27a.(V1x = V0a))) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27a^{A\_27c}).(\forall V2x \in A\_27c.((ap\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27c\ A\_27b\ A\_27a)\ V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27b^{A\_27c}).(\forall V1g \in (A\_27c^{A\_27a}).((ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ A\_27b\ A\_27c)\ V0f)\ (\lambda V2x \in A\_27a.(ap\ V1g\ V2x))) = (\lambda V3x \in A\_27a.(ap\ V0f\ (ap\ V1g\ V3x)))))) \tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow (\forall V0f \in ((A\_27b^{A\_27c})^{A\_27a}).(\forall V1g \in (A\_27c^{A\_27a}).((ap\ (ap\ (\mathsf{c\_2Ecombin\_2ES}\ A\_27a\ A\_27c\ A\_27b)\ V0f)\ (\lambda V2x \in A\_27a.(ap\ V1g\ V2x))) = (\lambda V3x \in A\_27a.(ap\ (ap\ V0f\ V3x)\ (ap\ V1g\ V3x)))))) \tag{40}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow (\forall V0f \in ((A\_27c^{A\_27b})^{A\_27a}).(\forall V1x \in A\_27b.(\forall V2y \in A\_27a.((ap\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2EC}\ A\_27a\ A\_27b\ A\_27c)\ V0f)\ V1x)\ V2y) = (ap\ (ap\ V0f\ V2y)\ V1x))))) \tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow (\forall V0P \in (A\_27a^{A\_27b}).(\forall V1f \in (A\_27b^{A\_27c}).(\forall V2v \in A\_27c.((ap\ V0P\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ A\_27c\ A\_27b)\ V1f)\ V2v)) = (ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ A\_27c\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27c\ A\_27a\ A\_27b)\ V0P)\ V1f))\ V2v))))) \tag{42}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (2^{A\_27a}).(\forall V1v \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ A\_27a\ 2)\ V0f)\ V1v)) \Leftrightarrow (p\ (ap\ (\mathsf{c\_2Ebool\_2E\_21}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2ES}\ A\_27a\ 2\ 2)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ (2^2)\ 2)\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E})\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ 2\ 2)\ \mathsf{c\_2Emarker\_2EAbbrev})\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2EC}\ A\_27a\ A\_27a\ 2)\ (\mathsf{c\_2Emin\_2E\_3D}\ A\_27a))\ V1v))))\ V0f)))))) \tag{43}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(((\mathsf{c\_2Earithmetic\_2EZERO} = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n)) \Leftrightarrow False) \wedge ((((ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n) = \mathsf{c\_2Earithmetic\_2EZERO}) \Leftrightarrow False) \wedge (((\mathsf{c\_2Earithmetic\_2EZERO} = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n)) \Leftrightarrow False) \wedge ((((ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n) = \mathsf{c\_2Earithmetic\_2EZERO}) \Leftrightarrow False) \wedge ((((ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)) \Leftrightarrow False) \wedge ((((ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)) \Leftrightarrow False) \wedge ((((ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)) \Leftrightarrow (V0n = V1m)))))))))))) \tag{44}$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p\ (ap\ V0P\ \mathsf{c\_2Earithmetic\_2EZERO})) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1n))))) \wedge (\forall V2n \in ty\_2Enum\_2Enum.((p\ (ap\ V0P\ V2n)) \Rightarrow (p\ (ap\ V0P\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V2n))))))) \Rightarrow (\forall V3n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ V3n))))) \tag{45}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \mathsf{c\_2Earithmetic\_2EZERO})\ V0n) = \mathsf{c\_2Earithmetic\_2EZERO}) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0n)\ \mathsf{c\_2Earithmetic\_2EZERO}) = \mathsf{c\_2Earithmetic\_2EZERO}) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ V1m) = (ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Enumeral\_2EiDUB}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0n)\ V1m)))\ V1m))) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ V1m) = (ap\ \mathsf{c\_2Enumeral\_2EiDUB}\ (ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0n)\ V1m))\ V1m))))))))) \tag{46}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ c\_2Earithmetic\_2EEVEN\ \mathsf{c\_2Earithmetic\_2EZERO}))\wedge ((p\ (ap\ c\_2Earithmetic\_2EEVEN\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n)))\wedge ((\neg(p\ (ap\ c\_2Earithmetic\_2EEVEN\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))))\wedge ((\neg(p\ (ap\ c\_2Earithmetic\_2EODD\ \mathsf{c\_2Earithmetic\_2EZERO})))\wedge((\neg(p\ (ap\ c\_2Earithmetic\_2EODD\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))))\wedge (p\ (ap\ c\_2Earithmetic\_2EODD\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n)))))))))) \tag{47}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1a \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Enumeral\_2Etexp\_\_help\ V0n)\ V1a) = (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1a)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ (ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))))) \tag{48}$$

Assume the following.

$$(((ap\ c\_2Enumeral\_2Eexactlog\ \mathsf{c\_2Earithmetic\_2EZERO}) = \mathsf{c\_2Earithmetic\_2EZERO})\wedge ((\forall V0n \in ty\_2Enum\_2Enum.((ap\ c\_2Enumeral\_2Eexactlog\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n)) = \mathsf{c\_2Earithmetic\_2EZERO}))\wedge(\forall V1n \in ty\_2Enum\_2Enum.((ap\ c\_2Enumeral\_2Eexactlog\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1n)) = (ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ (\lambda V2x \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Enum\_2Enum)\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ ty\_2Enum\_2Enum)\ V2x)\ \mathsf{c\_2Earithmetic\_2EZERO}))\ \mathsf{c\_2Earithmetic\_2EZERO})\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V2x))))\ (ap\ (ap\ c\_2Enumeral\_2Eonecount\ V1n)\ \mathsf{c\_2Earithmetic\_2EZERO})))))) \tag{49}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(((ap\ c\_2Enumeral\_2Eexactlog\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))\Rightarrow(V0n = (ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1m)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))))) \tag{50}$$

Assume the following.

$$(\forall V0x \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Earithmetic\_2EDIV2}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0x)) = V0x)) \tag{51}$$

Assume the following.

$$(((ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0}) \wedge (\forall V0m \in \mathit{ty\_2Enum\_2Enum}.((ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m)) = V0m))) \tag{52}$$

Theorem 1

$$(\forall V0n \in \mathit{ty\_2Enum\_2Enum}.(\forall V1x \in \mathit{ty\_2Enum\_2Enum}.(\forall V2y \in \mathit{ty\_2Enum\_2Enum}.(((ap\ (ap\ \mathit{c\_2Earithmetic\_2E\_2A}\ \mathsf{c\_2Earithmetic\_2EZERO})\ V0n) = \mathsf{c\_2Earithmetic\_2EZERO}) \wedge (((ap\ (ap\ \mathit{c\_2Earithmetic\_2E\_2A}\ V0n)\ \mathsf{c\_2Earithmetic\_2EZERO}) = \mathsf{c\_2Earithmetic\_2EZERO}) \wedge (((ap\ (ap\ \mathit{c\_2Earithmetic\_2E\_2A}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1x))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V2y)) = (ap\ (ap\ \mathsf{c\_2Enumeral\_2Einternal\_\_mult}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1x))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V2y))) \wedge (((ap\ (ap\ \mathit{c\_2Earithmetic\_2E\_2A}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1x))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V2y)) = (ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ \mathit{ty\_2Enum\_2Enum}\ \mathit{ty\_2Enum\_2Enum})\ (\lambda V3n \in \mathit{ty\_2Enum\_2Enum}.(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ \mathit{ty\_2Enum\_2Enum})\ (ap\ \mathit{c\_2Earithmetic\_2EODD}\ V3n))\ (ap\ (ap\ \mathit{c\_2Enumeral\_2Etexp\_\_help}\ (ap\ \mathsf{c\_2Earithmetic\_2EDIV2}\ V3n))\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1x))))\ (ap\ (ap\ \mathsf{c\_2Enumeral\_2Einternal\_\_mult}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1x))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V2y)))))\ (ap\ \mathit{c\_2Enumeral\_2Eexactlog}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V2y)))) \wedge (((ap\ (ap\ \mathit{c\_2Earithmetic\_2E\_2A}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1x))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V2y)) = (ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ \mathit{ty\_2Enum\_2Enum}\ \mathit{ty\_2Enum\_2Enum})\ (\lambda V4m \in \mathit{ty\_2Enum\_2Enum}.(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ \mathit{ty\_2Enum\_2Enum})\ (ap\ \mathit{c\_2Earithmetic\_2EODD}\ V4m))\ (ap\ (ap\ \mathit{c\_2Enumeral\_2Etexp\_\_help}\ (ap\ \mathsf{c\_2Earithmetic\_2EDIV2}\ V4m))\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V2y))))\ (ap\ (ap\ \mathsf{c\_2Enumeral\_2Einternal\_\_mult}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1x))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V2y)))))\ (ap\ \mathit{c\_2Enumeral\_2Eexactlog}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1x)))) \wedge ((ap\ (ap\ \mathit{c\_2Earithmetic\_2E\_2A}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1x))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V2y)) = (ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ \mathit{ty\_2Enum\_2Enum}\ \mathit{ty\_2Enum\_2Enum})\ (\lambda V5m \in \mathit{ty\_2Enum\_2Enum}.(ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ \mathit{ty\_2Enum\_2Enum}\ \mathit{ty\_2Enum\_2Enum})\ (\lambda V6n \in \mathit{ty\_2Enum\_2Enum}.(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ \mathit{ty\_2Enum\_2Enum})\ (ap\ \mathit{c\_2Earithmetic\_2EODD}\ V5m))\ (ap\ (ap\ \mathit{c\_2Enumeral\_2Etexp\_\_help}\ (ap\ \mathsf{c\_2Earithmetic\_2EDIV2}\ V5m))\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V2y))))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ \mathit{ty\_2Enum\_2Enum})\ (ap\ \mathit{c\_2Earithmetic\_2EODD}\ V6n))\ (ap\ (ap\ \mathit{c\_2Enumeral\_2Etexp\_\_help}\ (ap\ \mathsf{c\_2Earithmetic\_2EDIV2}\ V6n))\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1x))))\ (ap\ (ap\ \mathsf{c\_2Enumeral\_2Einternal\_\_mult}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1x))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V2y))))))\ (ap\ \mathit{c\_2Enumeral\_2Eexactlog}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V2y)))))\ (ap\ \mathit{c\_2Enumeral\_2Eexactlog}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1x))))))))))))$$