thm_2Enumeral_2EiSUB__THM (TMPzEdezuhxZLzdgkuMxebqxrKeFhoUzZ4M)

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Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{2}$$

Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27$

Definition 5 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{3}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{4}$$

Definition 6 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{5}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{6}$$

Definition 7 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 8 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 9 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 10 *We define* c_2Enumeral_2EiDUB *to be* $\lambda V0x \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Let $c\_2Enumeral\_2EiBIT\_\_cases : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Enumeral\_2EiBIT\_\_cases\ A\_27a \in ((((A\_27a^{(A\_27a^{ty\_2Enum\_2Enum})})^{(A\_27a^{ty\_2Enum\_2Enum})})^{A\_27a})^{ty\_2Enum\_2Enum}) \tag{7}$$

Definition 11 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 12 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 13 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 14 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 15 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0*.*

Let $c\_2Enumeral\_2EiSUB : \iota$ be given. Assume the following.

$$c\_2Enumeral\_2EiSUB \in (((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^2) \tag{8}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Enum\_2E0})\ V0m) = V0m) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ \mathsf{c\_2Enum\_2E0}) = V0m) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n)))))))) \tag{9}$$

Assume the following.

$$True \tag{10}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{11}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{12}$$

Assume the following.

$$(\forall V0t \in 2.(((True\vee(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\vee True)\Leftrightarrow True)\wedge(((False\vee(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\vee False)\Leftrightarrow(p\ V0t))\wedge(((p\ V0t)\vee(p\ V0t))\Leftrightarrow(p\ V0t)))))) \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((V0x = V0x)\Leftrightarrow True)) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1)\wedge((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2) = V1t2)))) \tag{15}$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p\ (ap\ V0P\ \mathsf{c\_2Enum\_2E0}))\wedge(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ V0P\ V1n))\Rightarrow(p\ (ap\ V0P\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))))))\Rightarrow(\forall V2n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ V2n))))) \tag{16}$$

Assume the following.

$$(((ap\ \mathsf{c\_2Enum\_2ESUC}\ \mathsf{c\_2Earithmetic\_2EZERO}) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))\wedge((\forall V0n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n)) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n)))\wedge(\forall V1n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1n)) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)))))) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow((\forall V0zf \in A\_27a.(\forall V1bf1 \in (A\_27a^{ty\_2Enum\_2Enum}).(\forall V2bf2 \in (A\_27a^{ty\_2Enum\_2Enum}).((ap\ (ap\ (ap\ (ap\ (c\_2Enumeral\_2EiBIT\_\_cases\ A\_27a)\ \mathsf{c\_2Earithmetic\_2EZERO})\ V0zf)\ V1bf1)\ V2bf2) = V0zf))))\wedge((\forall V3n \in ty\_2Enum\_2Enum.(\forall V4zf \in A\_27a.(\forall V5bf1 \in (A\_27a^{ty\_2Enum\_2Enum}).(\forall V6bf2 \in (A\_27a^{ty\_2Enum\_2Enum}).((ap\ (ap\ (ap\ (ap\ (c\_2Enumeral\_2EiBIT\_\_cases\ A\_27a)\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V3n))\ V4zf)\ V5bf1)\ V6bf2) = (ap\ V5bf1\ V3n))))))\wedge(\forall V7n \in ty\_2Enum\_2Enum.(\forall V8zf \in A\_27a.(\forall V9bf1 \in (A\_27a^{ty\_2Enum\_2Enum}).(\forall V10bf2 \in (A\_27a^{ty\_2Enum\_2Enum}).((ap\ (ap\ (ap\ (ap\ (c\_2Enumeral\_2EiBIT\_\_cases\ A\_27a)\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V7n))\ V8zf)\ V9bf1)\ V10bf2) = (ap\ V10bf2\ V7n)))))))) \tag{18}$$

Assume the following.

$$((\forall V0b \in 2.(\forall V1x \in ty\_2Enum\_2Enum.((ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ V0b)\ \mathsf{c\_2Earithmetic\_2EZERO})\ V1x) = \mathsf{c\_2Earithmetic\_2EZERO})))\land ((\forall V2b \in 2.(\forall V3n \in ty\_2Enum\_2Enum.(\forall V4x \in ty\_2Enum\_2Enum.((ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ V2b)\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V3n))\ V4x) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Enum\_2Enum)\ V2b)\ (ap\ (ap\ (ap\ (ap\ (c\_2Enumeral\_2EiBIT\_\_cases\ ty\_2Enum\_2Enum)\ V4x)\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V3n))\ (\lambda V5m \in ty\_2Enum\_2Enum.(ap\ \mathsf{c\_2Enumeral\_2EiDUB}\ (ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ \mathsf{c\_2Ebool\_2ET})\ V3n)\ V5m))))\ (\lambda V6m \in ty\_2Enum\_2Enum.(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ \mathsf{c\_2Ebool\_2EF})\ V3n)\ V6m)))))\ (ap\ (ap\ (ap\ (ap\ (c\_2Enumeral\_2EiBIT\_\_cases\ ty\_2Enum\_2Enum)\ V4x)\ (ap\ \mathsf{c\_2Enumeral\_2EiDUB}\ V3n))\ (\lambda V7m \in ty\_2Enum\_2Enum.(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ \mathsf{c\_2Ebool\_2EF})\ V3n)\ V7m))))\ (\lambda V8m \in ty\_2Enum\_2Enum.(ap\ \mathsf{c\_2Enumeral\_2EiDUB}\ (ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ \mathsf{c\_2Ebool\_2EF})\ V3n)\ V8m))))))))))\land(\forall V9b \in 2.(\forall V10n \in ty\_2Enum\_2Enum.(\forall V11x \in ty\_2Enum\_2Enum.((ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ V9b)\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V10n))\ V11x) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Enum\_2Enum)\ V9b)\ (ap\ (ap\ (ap\ (ap\ (c\_2Enumeral\_2EiBIT\_\_cases\ ty\_2Enum\_2Enum)\ V11x)\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V10n))\ (\lambda V12m \in ty\_2Enum\_2Enum.(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ \mathsf{c\_2Ebool\_2ET})\ V10n)\ V12m))))\ (\lambda V13m \in ty\_2Enum\_2Enum.(ap\ \mathsf{c\_2Enumeral\_2EiDUB}\ (ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ \mathsf{c\_2Ebool\_2ET})\ V10n)\ V13m)))))\ (ap\ (ap\ (ap\ (ap\ (c\_2Enumeral\_2EiBIT\_\_cases\ ty\_2Enum\_2Enum)\ V11x)\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V10n))\ (\lambda V14m \in ty\_2Enum\_2Enum.(ap\ \mathsf{c\_2Enumeral\_2EiDUB}\ (ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ \mathsf{c\_2Ebool\_2ET})\ V10n)\ V14m))))\ (\lambda V15m \in ty\_2Enum\_2Enum.(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ \mathsf{c\_2Ebool\_2EF})\ V10n)\ V15m)))))))))))) \tag{19}$$

Theorem 1

$$(\forall V0x \in ty_2Enum_2Enum.(\forall V1b \in 2.(\forall V2n \in ty_2Enum_2Enum.
(\forall V3m \in ty_2Enum_2Enum.(((ap\ (ap\ (ap\ c_2Enumeral_2EiSUB
V1b)\ \mathsf{c_2Earithmetic_2EZERO})\ V0x) = \mathsf{c_2Earithmetic_2EZERO})\wedge(
((ap\ (ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2ET})\ V2n)\ \mathsf{c_2Earithmetic_2EZERO}) =
V2n)\wedge(((ap\ (ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2EF})\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}
V2n))\ \mathsf{c_2Earithmetic_2EZERO}) = (ap\ \mathsf{c_2Enumeral_2EiDUB}\ V2n))\wedge
(((ap\ (ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2ET})\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}
V2n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V3m)) = (ap\ \mathsf{c_2Enumeral_2EiDUB}
(ap\ (ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2ET})\ V2n)\ V3m)))\wedge(((ap
(ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2EF})\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}
V2n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V3m)) = (ap\ \mathsf{c_2Earithmetic_2EBIT1}
(ap\ (ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2EF})\ V2n)\ V3m)))\wedge(((ap
(ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2ET})\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}
V2n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V3m)) = (ap\ \mathsf{c_2Earithmetic_2EBIT1}
(ap\ (ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2EF})\ V2n)\ V3m)))\wedge(((ap
(ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2EF})\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}
V2n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V3m)) = (ap\ \mathsf{c_2Enumeral_2EiDUB}
(ap\ (ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2EF})\ V2n)\ V3m)))\wedge(((ap
(ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2EF})\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}
V2n))\ \mathsf{c_2Earithmetic_2EZERO}) = (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V2n))\wedge
(((ap\ (ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2ET})\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}
V2n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V3m)) = (ap\ \mathsf{c_2Earithmetic_2EBIT1}
(ap\ (ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2ET})\ V2n)\ V3m)))\wedge(((ap
(ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2EF})\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}
V2n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V3m)) = (ap\ \mathsf{c_2Enumeral_2EiDUB}
(ap\ (ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2ET})\ V2n)\ V3m)))\wedge(((ap
(ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2ET})\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}
V2n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V3m)) = (ap\ \mathsf{c_2Enumeral_2EiDUB}
(ap\ (ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2ET})\ V2n)\ V3m)))\wedge((ap
(ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2EF})\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}
V2n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V3m)) = (ap\ \mathsf{c_2Earithmetic_2EBIT1}
(ap\ (ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2EF})\ V2n)\ V3m))))))))))))))))))$$