

thm_2Enumeral_2Einternal__mult__characterisation
(TMEp-
Nyp5QsP66xvL3E64sJM2XNzkCoooVC8)

October 26, 2020

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 3 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define c_2Ebool_2E21 to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A-27a}))\ (\lambda V1P \in 2.V1P))\ (\lambda V2P \in 2.V2P))\ (\lambda V3P \in 2.V3P))$

Definition 5 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (c_2Emin_2E3D\ (2^2)))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 6 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B$

Definition 7 We define $c_2Enumeral_2EiDUB$ to be $\lambda V0x \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B$

Definition 8 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 9 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B$

Definition 10 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Definition 11 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 12 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Definition 13 We define $c_2Enumeral_2Einternal_mult$ to be $c_2Earithmetic_2E_2A$.

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum.(\forall V1m \in ty_2Enum_2Enum.(\\ & ((ap (ap c_2Earithmetic_2E_2A c_2Earithmetic_2EZERO) V0n) = c_2Earithmetic_2EZERO) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A V0n) c_2Earithmetic_2EZERO) = \\ & c_2Earithmetic_2EZERO) \wedge ((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2EBIT1 \\ & V0n)) V1m) = (ap c_2Enumeral_2EiZ (ap (ap c_2Earithmetic_2E_2B \\ & (ap c_2Enumeral_2EiDUB (ap (ap c_2Earithmetic_2E_2A V0n) V1m))) \\ & V1m))) \wedge ((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2EBIT2 \\ & V0n)) V1m) = (ap c_2Enumeral_2EiDUB (ap c_2Enumeral_2EiZ (ap (ap \\ & c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0n) V1m)) \\ & V1m)))))))))) \end{aligned} \quad (8)$$

Theorem 1

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum.(\forall V1m \in ty_2Enum_2Enum.(\\ & ((ap (ap c_2Enumeral_2Einternal_mult c_2Earithmetic_2EZERO) \\ & V0n) = c_2Earithmetic_2EZERO) \wedge (((ap (ap c_2Enumeral_2Einternal_mult \\ & V0n) c_2Earithmetic_2EZERO) = c_2Earithmetic_2EZERO) \wedge (((ap \\ & (ap c_2Enumeral_2Einternal_mult (ap c_2Earithmetic_2EBIT1 \\ & V0n)) V1m) = (ap c_2Enumeral_2EiZ (ap (ap c_2Earithmetic_2E_2B \\ & (ap c_2Enumeral_2EiDUB (ap (ap c_2Enumeral_2Einternal_mult \\ & V0n) V1m))) V1m))) \wedge ((ap (ap c_2Enumeral_2Einternal_mult (ap \\ & c_2Earithmetic_2EBIT2 V0n)) V1m) = (ap c_2Enumeral_2EiDUB (ap \\ & c_2Enumeral_2EiZ (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Enumeral_2Einternal_mult \\ & V0n) V1m)) V1m)))))))))) \end{aligned}$$