

thm_2Enumeral_2Enumeral_MIN
(TMHRa2D1TUUXUZYPDWXTRBX9R2gN4rPNgMb)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

nonempty *ty_2Enum_2Enum* (1)

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \rightarrow \iota$.

Definition 2 We define c_Ebool_ET to be $(ap \ (ap \ (c_Emin_3D \ (2^2)) \ (\lambda V0x \in 2.V0x)) \ (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^A \rightarrow 27a)).(ap\ (ap\ (c_2Emin_2E_3D\ (2^A \rightarrow 27a)\ V0) P) a)$

Definition 4 We define `c_2Earithmetic_2ENUMERAL` to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Definition 5 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 7 We define $c_2Eb0l_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Eb0l_2EF))$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap(c_2Ebool_2E_21 2))(\lambda V2t \in 2.$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (2)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^\omega)^\omega \quad (3)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (4)$$

Definition 9 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ ($

Definition 10 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\lambda x. x \in A \wedge P x) \text{ else } \perp$ of type $\iota \Rightarrow \iota$.

Definition 11 We define $c_2Ebool_2E_3F$ to be $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A-27a}). (ap V0P (ap (c_2Emin_2E_40 V0a) a)))$

Definition 12 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (V1t1 = V2t2))))$

Definition 13 We define c_2Ebool_2ECOND to be $\lambda A. \lambda a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (V1t1 = V2t2))))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (5)$$

Definition 14 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Definition 15 We define $c_2Earithmetic_2EMIN$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (V1t1 = V2t2))))$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (((ap (ap c_2Earithmetic_2EMIN V0n) c_2Enum_2E0) = c_2Enum_2E0) \wedge ((ap (ap c_2Earithmetic_2EMIN c_2Enum_2E0) V0n) = c_2Enum_2E0))) \quad (6)$$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (8)$$

Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (9)$$

Theorem 1

$$\begin{aligned} & (\forall V0x \in ty_2Enum_2Enum. (\forall V1y \in ty_2Enum_2Enum. \\ & ((ap (ap c_2Earithmetic_2EMIN c_2Enum_2E0) V0x) = c_2Enum_2E0) \wedge \\ & (((ap (ap c_2Earithmetic_2EMIN V0x) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\ & ((ap (ap c_2Earithmetic_2EMIN (ap c_2Earithmetic_2ENUMERAL V0x)) \\ & (ap c_2Earithmetic_2ENUMERAL V1y)) = (ap c_2Earithmetic_2ENUMERAL \\ & (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) (ap (ap c_2Eprim_rec_2E_3C \\ & V0x) V1y)) V0x) V1y))))))) \end{aligned}$$