

thm_2Enumeral_2Enumeral__div2
(TMQwc7U18b1qdGt3SSWavDCv63QwXQPPq2k)

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{6}$$

Definition 7 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EBIT2 V0n) V0n)$.

Definition 8 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (7)$$

Definition 9 We define $c_2Earithmetic_2EDIV2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EDIV2 V0n) V0n)$.

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (8)$$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (9)$$

Definition 10 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EBIT1 V0n) V0n)$.

Definition 11 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 12 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 13 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21 2) V0t)$.

Definition 14 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t) V1t2) V0t1))$.

Definition 15 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \Rightarrow p x))$ of type $\iota \Rightarrow \iota$.

Definition 16 We define $c_2Ebool_2E_3F$ to be $\lambda A.^{27a} : \iota.(\lambda V0P \in (2^A)^{27a}).(ap V0P (ap (c_2Emin_2E_40 V0P) V0P))$.

Definition 17 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap (ap c_2Eprim_rec_2E_3C V0m) V1n)$.

Definition 18 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t) V1t2) V0t1))$.

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum. \\ & ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge (((ap (\\ & ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2B \\ & (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B \\ & V0m) V1n)))) \wedge ((ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Enum_2ESUC \\ & V1n)) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n)))))))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V0m) \\
& (ap (ap c_2Earithmetic_2E_2B V1n) V2p)) = (ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p))))))
\end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (ap (ap c_2Earithmetic_2E_2A V0m) V1n) = (ap (ap c_2Earithmetic_2E_2A \\
& \quad V1n) V0m))))
\end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A (\\
& ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) \\
& V0n) = (ap (ap c_2Earithmetic_2E_2B V0n) V0n)))
\end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
& c_2Enum_2E0) V0n)) \Rightarrow ((ap (ap c_2Earithmetic_2EDIV c_2Enum_2E0) \\
& \quad V0n) = c_2Enum_2E0)))
\end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (p (ap (ap c_2Eprim_rec_2E_3C V0r) V1n)) \Rightarrow ((ap (ap c_2Earithmetic_2EDIV \\
& \quad V0r) V1n) = c_2Enum_2E0))))
\end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
& c_2Enum_2E0) V0n)) \Rightarrow (\forall V1x \in ty_2Enum_2Enum. (\forall V2r \in \\
& ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EDIV (ap (ap c_2Earithmetic_2E_2B \\
& (ap (ap c_2Earithmetic_2E_2A V1x) V0n)) V2r)) V0n) = (ap (ap c_2Earithmetic_2E_2B \\
& \quad V1x) (ap (ap c_2Earithmetic_2EDIV V2r) V0n)))))))
\end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
& c_2Enum_2E0) V0n)) \Rightarrow (((ap (ap c_2Earithmetic_2EDIV V0n) V0n) = \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \wedge \\
& ((ap (ap c_2Earithmetic_2EMOD V0n) V0n) = c_2Enum_2E0))))
\end{aligned} \tag{17}$$

Assume the following.

$$\text{True} \tag{18}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (19)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in (2^{A_27a}).((\forall V2x \in A_27a.((p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((\forall V3x \in A_27a.(p (ap V0P V3x))) \wedge (\forall V4x \in A_27a.(p (ap V1Q V4x))))))) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A_27a}).((p V0P) \wedge (\forall V2x \in A_27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A_27a.((p V0P) \wedge (p (ap V1Q V3x)))))) \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A_27a}).((\forall V2x \in A_27a.((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A_27a.(p (ap V1P V3x))) \vee (p V0Q)))))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (27)$$

Assume the following.

$$\begin{aligned}
& (((ap\ c._2Enum_2ESUC\ c._2Earithmetic_2EZERO) = (ap\ c._2Earithmetic_2EBIT1 \\
& \quad c._2Earithmetic_2EZERO)) \wedge ((\forall V0n \in ty._2Enum._2Enum. ((ap \\
& \quad c._2Enum_2ESUC\ (ap\ c._2Earithmetic_2EBIT1\ V0n)) = (ap\ c._2Earithmetic_2EBIT2 \\
& \quad V0n))) \wedge (\forall V1n \in ty._2Enum._2Enum. ((ap\ c._2Enum_2ESUC\ (ap\ c._2Earithmetic_2EBIT2 \\
& \quad V1n)) = (ap\ c._2Earithmetic_2EBIT1\ (ap\ c._2Enum_2ESUC\ V1n))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty._2Enum._2Enum. (\forall V1m \in ty._2Enum._2Enum. (\\
& ((p\ (ap\ (ap\ c._2Eprim_rec._2E_3C\ c._2Earithmetic_2EZERO)\ (ap\ c._2Earithmetic_2EBIT1 \\
& \quad V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ c._2Eprim_rec._2E_3C\ c._2Earithmetic_2EZERO) \\
& \quad (ap\ c._2Earithmetic_2EBIT2\ V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ c._2Eprim_rec._2E_3C \\
& \quad V0n)\ c._2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ c._2Eprim_rec._2E_3C \\
& \quad (ap\ c._2Earithmetic_2EBIT1\ V0n))\ (ap\ c._2Earithmetic_2EBIT1\ V1m))) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ c._2Eprim_rec._2E_3C\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c._2Eprim_rec._2E_3C \\
& \quad (ap\ c._2Earithmetic_2EBIT2\ V0n))\ (ap\ c._2Earithmetic_2EBIT2\ V1m))) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ c._2Eprim_rec._2E_3C\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c._2Eprim_rec._2E_3C \\
& \quad (ap\ c._2Earithmetic_2EBIT1\ V0n))\ (ap\ c._2Earithmetic_2EBIT2\ V1m))) \Leftrightarrow \\
& \quad (\neg(p\ (ap\ (ap\ c._2Eprim_rec._2E_3C\ V1m)\ V0n))) \wedge ((p\ (ap\ (ap\ c._2Eprim_rec._2E_3C \\
& \quad (ap\ c._2Earithmetic_2EBIT2\ V0n))\ (ap\ c._2Earithmetic_2EBIT1\ V1m))) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ c._2Eprim_rec._2E_3C\ V0n)\ V1m))))))
\end{aligned} \tag{29}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{30}$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{31}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))
\end{aligned} \tag{33}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\
& \quad (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\
& \quad p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& \quad ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{39}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{40}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{41}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \tag{42}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{43}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \tag{44}$$

Theorem 1

$$\begin{aligned}
& (((ap \text{ c_2Earithmetic_2EDIV2 } c_2Enum_2E0) = c_2Enum_2E0) \wedge ((\forall V0n \in \\
& ty_2Enum_2Enum. ((ap \text{ c_2Earithmetic_2EDIV2 } (ap \text{ c_2Earithmetic_2ENUMERAL } \\
& (ap \text{ c_2Earithmetic_2EBIT1 } V0n))) = (ap \text{ c_2Earithmetic_2ENUMERAL } \\
& V0n))) \wedge (\forall V1n \in ty_2Enum_2Enum. ((ap \text{ c_2Earithmetic_2EDIV2 } \\
& (ap \text{ c_2Earithmetic_2ENUMERAL } (ap \text{ c_2Earithmetic_2EBIT2 } V1n))) = \\
& (ap \text{ c_2Earithmetic_2ENUMERAL } (ap \text{ c_2Enum_2ESUC } V1n))))))
\end{aligned}$$