

thm_2Enumeral_2Enumeral_sub (TMb- dRewHfnEtAcxLhF8EZgjLxD3dsFLKQ4G)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 5 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 6 We define $c_2Ebool_2E_3F$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{2}$$

Definition 7 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{3}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (4)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (5)$$

Definition 11 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 12 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 13 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 14 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 15 We define $c_2Enumeral_2EiDUB$ to be $\lambda V0x \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B$

Let $c_2Enumeral_2EiBIT_cases : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Enumeral_2EiBIT_cases\ A_27a \in \\ (((A_27a^{(A_27a^{ty_2Enum_2Enum})})^{(A_27a^{ty_2Enum_2Enum})})^{A_27a})^{ty_2Enum_2Enum}) \quad (7)$$

Definition 16 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Let $c_2Enumeral_2EiSUB : \iota$ be given. Assume the following.

$$c_2Enumeral_2EiSUB \in (((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^2) \quad (8)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (9)$$

Definition 17 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 18 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B$

Definition 19 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B$

Definition 20 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Assume the following.

$$\begin{aligned}
& ((\forall V0m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& c_2Enum_2E0) V0m) = c_2Enum_2E0)) \wedge (\forall V1m \in ty_2Enum_2Enum. \\
& (\forall V2n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D (\\
& ap c_2Enum_2ESUC V1m)) V2n) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) \\
& (ap (ap c_2Eprim_rec_2E_3C V1m) V2n)) c_2Enum_2E0) (ap c_2Enum_2ESUC \\
& (ap (ap c_2Earithmetic_2E_2D V1m) V2n)))))))
\end{aligned} \tag{10}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\
& ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge (((ap (\\
& ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2B \\
& (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B \\
& V0m) V1n))) \wedge ((ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Enum_2ESUC \\
& V1n)) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n)))))))
\end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\
& \forall V2p \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B V0m) \\
& (ap (ap c_2Earithmetic_2E_2B V1n) V2p)) = (ap (ap c_2Earithmetic_2E_2B \\
& (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p))))
\end{aligned} \tag{12}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((V0m = c_2Enum_2E0) \vee (\exists V1n \in ty_2Enum_2Enum.(V0m = (ap c_2Enum_2ESUC V1n)))))) \tag{13}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\
& \forall V2p \in ty_2Enum_2Enum.(((p (ap (ap c_2Eprim_rec_2E_3C \\
& V0m) V1n)) \wedge (p (ap (ap c_2Eprim_rec_2E_3C V1n) V2p))) \Rightarrow (p (ap (ap \\
& c_2Eprim_rec_2E_3C V0m) V2p))))))
\end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\
& (p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Enum_2ESUC V0m)) (ap c_2Enum_2ESUC \\
& V1n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n))))
\end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum.(\forall V1m \in ty_2Enum_2Enum.(\\
& (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V0n)) (ap \\
& c_2Enum_2ESUC V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V0n) \\
& V1m))))
\end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & \neg((p (ap (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)) \wedge (p (ap (ap c_2Eprim_rec_2E_3C \\ & V1n) (ap c_2Enum_2ESUC V0m)))))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\ & (ap c_2Enum_2ESUC V0m)) V1n)))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum. (p (ap (ap c_2Earithmetic_2E_3C_3D \\ & c_2Enum_2E0) V0n))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum. (\neg(p (ap (ap c_2Earithmetic_2E_3C_3D \\ & (ap c_2Enum_2ESUC V0n)) c_2Enum_2E0)))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (\neg(p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\ & V1n) V0m)))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (((ap (ap c_2Earithmetic_2E_2D \\ & c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge ((ap (ap c_2Earithmetic_2E_2D \\ & V0m) c_2Enum_2E0) = V0m))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\ & (ap (ap c_2Earithmetic_2E_2D (ap c_2Enum_2ESUC V0n)) (ap c_2Enum_2ESUC \\ & V1m)) = (ap (ap c_2Earithmetic_2E_2D V0n) V1m)))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & ((ap (ap c_2Earithmetic_2E_2D V0m) V1n) = c_2Enum_2E0) \Leftrightarrow (p (ap (\\ & ap c_2Earithmetic_2E_3C_3D V0m) V1n)))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & \forall V2p \in ty_2Enum_2Enum. (((p (ap (ap c_2Eprim_rec_2E_3C \\ & V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))) \Rightarrow (p (\\ & ap (ap c_2Eprim_rec_2E_3C V0m) V2p)))))) \end{aligned} \quad (25)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V0m))) \quad (26)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)) \Rightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)))) \quad (27)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\forall V1m \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D V0n) V1m)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1m) V0n))) \Rightarrow (V0n = V1m)))) \quad (28)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(p (ap (ap c_2Eprim_rec_2E_3C V0m) (ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Enum_2ESUC V1n)))))) \quad (29)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2D V1n) V2p)) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p)) V0m) (ap (ap c_2Earithmetic_2E_2D (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p)))))) \quad (30)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2D V0m) V1n)) V2p) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) V2p) (ap (ap c_2Earithmetic_2E_2D (ap (ap c_2Earithmetic_2E_2B V0m) V2p)) V1n)))))) \quad (31)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D (ap (ap c_2Earithmetic_2E_2D V0m) V1n)) V2p) = (ap (ap c_2Earithmetic_2E_2D V0m) (ap (ap c_2Earithmetic_2E_2B V1n) V2p)))))) \quad (32)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2D\ V0m)\ V1n)) = (ap \\
& \quad (ap\ (ap\ (c_2Ebool_2ECOND\ ty_2Enum_2Enum)\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D \\
& \quad V0m)\ V1n))\ (ap\ c_2Enum_2ESUC\ c_2Enum_2E0))\ (ap\ (ap\ c_2Earithmetic_2E_2D \\
& \quad (ap\ c_2Enum_2ESUC\ V0m))\ V1n))))))
\end{aligned} \tag{33}$$

Assume the following.

$$True \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\
& \quad V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2))))))
\end{aligned} \tag{35}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \tag{36}$$

Assume the following.

$$(\forall V0t \in 2. ((p\ V0t) \vee (\neg(p\ V0t)))) \tag{37}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\
& \quad A_27a. (p\ V0t)) \Leftrightarrow (p\ V0t)))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\
& \quad (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& \quad (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge \\
& \quad (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee \\
& \quad (p\ V0t)) \Leftrightarrow (p\ V0t))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\
& \quad True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\
& \quad (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
& \quad ((\neg False) \Leftrightarrow True))))
\end{aligned} \tag{42}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (43)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (44)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in \\ & A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\ & V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF) \\ & V0t1)\ V1t2) = V1t2)))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0b \in 2. (\forall V1f \in (A_27b^{A_27a}). (\forall V2g \in (A_27b^{A_27a}). \\ & (\forall V3x \in A_27a. ((ap\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (A_27b^{A_27a}) \\ & V0b)\ V1f)\ V2g)\ V3x) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27b)\ V0b)\ (ap \\ & V1f\ V3x))\ (ap\ V2g\ V3x)))))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0f \in (A_27b^{A_27a}). (\forall V1b \in 2. (\forall V2x \in A_27a. \\ & (\forall V3y \in A_27a. ((ap\ V0f\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\ & V1b)\ V2x)\ V3y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27b)\ V1b)\ (ap\ V0f \\ & V2x))\ (ap\ V0f\ V3y)))))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\ & 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\
& (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\
& (\forall V5y_27 \in A_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\
& ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap (ap (ap (c_2Ebool_2ECOND A_27a) \\
& V0P) V2x) V4y) = (ap (ap (ap (c_2Ebool_2ECOND A_27a) V1Q) V3x_27) \\
& V5y_27)))))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty_2Enum_2Enum}). (((p (ap V0P c_2Enum_2E0)) \wedge \\
& (\forall V1n \in ty_2Enum_2Enum. ((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c_2Enum_2ESUC \\
& V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum. (p (ap V0P V2n))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (((ap c_2Enum_2ESUC c_2Earithmetic_2EZERO) = (ap c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO)) \wedge (\forall V0n \in ty_2Enum_2Enum. ((ap \\
& c_2Enum_2ESUC (ap c_2Earithmetic_2EBIT1 V0n)) = (ap c_2Earithmetic_2EBIT2 \\
& V0n))) \wedge (\forall V1n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC (ap c_2Earithmetic_2EBIT2 \\
& V1n)) = (ap c_2Earithmetic_2EBIT1 (ap c_2Enum_2ESUC V1n))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) (ap c_2Earithmetic_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& (ap c_2Earithmetic_2EBIT2 V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& V0n) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT1 V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT2 V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT1 V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow \\
& (\neg(p (ap (ap c_2Eprim_rec_2E_3C V1m) V0n)))) \wedge ((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT2 V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m)))))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D c_2Earithmetic_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Earithmetic_2EBIT2 V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (\neg (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V1m) V0n)))) \wedge ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))))))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow ((\forall V0zf \in A_27a. (\forall V1bf1 \in \\
& (A_27a^{ty_2Enum_2Enum}). (\forall V2bf2 \in (A_27a^{ty_2Enum_2Enum}). \\
& ((ap (ap (ap (ap (c_2Enumeral_2EiBIT_cases A_27a) c_2Earithmetic_2EZERO) \\
& V0zf) V1bf1) V2bf2) = V0zf)))) \wedge ((\forall V3n \in ty_2Enum_2Enum. \\
& (\forall V4zf \in A_27a. (\forall V5bf1 \in (A_27a^{ty_2Enum_2Enum}). \\
& (\forall V6bf2 \in (A_27a^{ty_2Enum_2Enum}). ((ap (ap (ap (ap (c_2Enumeral_2EiBIT_cases \\
& A_27a) (ap c_2Earithmetic_2EBIT1 V3n)) V4zf) V5bf1) V6bf2) = (ap \\
& V5bf1 V3n)))))) \wedge ((\forall V7n \in ty_2Enum_2Enum. (\forall V8zf \in \\
& A_27a. (\forall V9bf1 \in (A_27a^{ty_2Enum_2Enum}). (\forall V10bf2 \in \\
& (A_27a^{ty_2Enum_2Enum}). ((ap (ap (ap (ap (c_2Enumeral_2EiBIT_cases \\
& A_27a) (ap c_2Earithmetic_2EBIT2 V7n)) V8zf) V9bf1) V10bf2) = (\\
& ap V10bf2 V7n))))))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0b \in 2. (\forall V1x \in ty_2Enum_2Enum. ((ap (ap (ap c_2Enumeral_2EiSUB \\
& \quad V0b) c_2Earithmic_2EZERO) V1x) = c_2Earithmic_2EZERO))) \wedge \\
& \quad ((\forall V2b \in 2. (\forall V3n \in ty_2Enum_2Enum. (\forall V4x \in \\
& ty_2Enum_2Enum. ((ap (ap (ap c_2Enumeral_2EiSUB V2b) (ap c_2Earithmic_2EBIT1 \\
& \quad V3n)) V4x) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) V2b) (\\
& \quad ap (ap (ap (ap (c_2Enumeral_2EiBIT_cases ty_2Enum_2Enum) V4x) \\
& \quad (ap c_2Earithmic_2EBIT1 V3n)) (\lambda V5m \in ty_2Enum_2Enum. (ap \\
& \quad c_2Enumeral_2EiDUB (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) \\
& \quad V3n) V5m)))) (\lambda V6m \in ty_2Enum_2Enum. (ap c_2Earithmic_2EBIT1 \\
& \quad (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) V3n) V6m)))))) (ap \\
& \quad (ap (ap (ap (c_2Enumeral_2EiBIT_cases ty_2Enum_2Enum) V4x) (\\
& \quad ap c_2Enumeral_2EiDUB V3n)) (\lambda V7m \in ty_2Enum_2Enum. (ap c_2Earithmic_2EBIT1 \\
& \quad (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) V3n) V7m)))) (\lambda V8m \in \\
& \quad ty_2Enum_2Enum. (ap c_2Enumeral_2EiDUB (ap (ap (ap c_2Enumeral_2EiSUB \\
& \quad c_2Ebool_2EF) V3n) V8m)))))) \wedge (\forall V9b \in 2. (\forall V10n \in \\
& ty_2Enum_2Enum. (\forall V11x \in ty_2Enum_2Enum. ((ap (ap (ap c_2Enumeral_2EiSUB \\
& \quad V9b) (ap c_2Earithmic_2EBIT2 V10n)) V11x) = (ap (ap (ap (c_2Ebool_2ECOND \\
& \quad ty_2Enum_2Enum) V9b) (ap (ap (ap (ap (c_2Enumeral_2EiBIT_cases \\
& \quad ty_2Enum_2Enum) V11x) (ap c_2Earithmic_2EBIT2 V10n)) (\lambda V12m \in \\
& \quad ty_2Enum_2Enum. (ap c_2Earithmic_2EBIT1 (ap (ap (ap c_2Enumeral_2EiSUB \\
& \quad c_2Ebool_2ET) V10n) V12m)))) (\lambda V13m \in ty_2Enum_2Enum. (ap c_2Enumeral_2EiDUB \\
& \quad (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V10n) V13m)))))) (\\
& \quad ap (ap (ap (ap (c_2Enumeral_2EiBIT_cases ty_2Enum_2Enum) V11x) \\
& \quad (ap c_2Earithmic_2EBIT1 V10n)) (\lambda V14m \in ty_2Enum_2Enum. \\
& \quad (ap c_2Enumeral_2EiDUB (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) \\
& \quad V10n) V14m)))) (\lambda V15m \in ty_2Enum_2Enum. (ap c_2Earithmic_2EBIT1 \\
& \quad (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) V10n) V15m))))))))) \\
& \hspace{15em} (57)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0P \in (2^{ty_2Enum_2Enum}). (((p (ap V0P c_2Earithmic_2EZERO)) \wedge \\
& ((\forall V1n \in ty_2Enum_2Enum. ((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c_2Earithmic_2EBIT1 \\
& \quad V1n)))))) \wedge (\forall V2n \in ty_2Enum_2Enum. ((p (ap V0P V2n)) \Rightarrow (p (ap \\
& \quad V0P (ap c_2Earithmic_2EBIT2 V2n)))))) \Rightarrow (\forall V3n \in ty_2Enum_2Enum. \\
& \quad (p (ap V0P V3n)))))) \\
& \hspace{15em} (58)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (ap c_2Enum_2ESUC V0m) = (ap c_2Enum_2ESUC V1n)) \Leftrightarrow (V0m = V1n))) \\
& \hspace{15em} (59)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum. (\neg (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V0n) V0n)))) \\
& \hspace{15em} (60)
\end{aligned}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg(p (ap (ap c_2Eprim_rec_2E_3C V0n) c_2Enum_2E0)))) \quad (61)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Enum_2ESUC V0n)))) \quad (62)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (63)$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (64)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (65)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (66)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (67)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (68)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (69)$$

Theorem 1

$$(\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V0n) V1m)) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) (ap (ap c_2Eprim_rec_2E_3C V1m) V0n)) (ap c_2Earithmetic_2ENUMERAL (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V0n) V1m))) c_2Enum_2E0))))$$