

# thm\_2Enumeral\_2Etexp\_help0 (TM-Riv1ePi9MBp1D3QQyxmFQgDZtGpdVX5nz)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (2)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (3)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (4)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$ .

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})) (\lambda V1P \in 2.V1P)) (\lambda V2P \in 2.V2P)))$ .

**Definition 4** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap c\_2Enum\_2EABS\_num m)$ .

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (5)$$

**Definition 5** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

**Definition 6** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 7** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p \Rightarrow p Q)$  of type  $\iota$ .

**Definition 8** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap(c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.$

**Definition 9** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap(c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$

**Definition 10** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap(ap(c\_2Earithmetic\_2E\_2B$

Let  $c\_2Earithmetic\_2EEEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (7)$$

**Definition 11** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

**Definition 12** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap(ap(c\_2Earithmetic\_2E\_2B$

**Definition 13** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (8)$$

Let  $c\_2Enumeral\_2Etexp\_help : \iota$  be given. Assume the following.

$$c\_2Enumeral\_2Etexp\_help \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (9)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(( \\ & ((ap(ap(c\_2Earithmetic\_2E\_2B c\_2Enum\_2E0) V0m) = V0m) \wedge (((ap( \\ & ap(c\_2Earithmetic\_2E\_2B V0m) c\_2Enum\_2E0) = V0m) \wedge (((ap(ap(c\_2Earithmetic\_2E\_2B \\ & (ap(c\_2Enum\_2ESUC V0m)) V1n) = (ap(c\_2Enum\_2ESUC (ap(ap(c\_2Earithmetic\_2E\_2B \\ & V0m) V1n))) \wedge ((ap(ap(c\_2Earithmetic\_2E\_2B V0m) (ap(c\_2Enum\_2ESUC \\ & V1n)) = (ap(c\_2Enum\_2ESUC (ap(ap(c\_2Earithmetic\_2E\_2B V0m) V1n))))))))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(( \\ & ((ap(ap(c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge \\ & (((ap(ap(c\_2Earithmetic\_2E\_2A V0m) c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge \\ & (((ap(ap(c\_2Earithmetic\_2E\_2A (ap(c\_2Earithmetic\_2ENUMERAL \\ & (ap(c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0m) = V0m) \wedge \\ & (((ap(ap(c\_2Earithmetic\_2E\_2A V0m) (ap(c\_2Earithmetic\_2ENUMERAL \\ & (ap(c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = V0m) \wedge \\ & ((ap(ap(c\_2Earithmetic\_2E\_2A (ap(c\_2Enum\_2ESUC V0m)) V1n) = (ap \\ & (ap(c\_2Earithmetic\_2E\_2B (ap(ap(c\_2Earithmetic\_2E\_2A V0m) V1n)) \\ & V1n)) \wedge ((ap(ap(c\_2Earithmetic\_2E\_2A V0m) (ap(c\_2Enum\_2ESUC V1n)) = \\ & (ap(ap(c\_2Earithmetic\_2E\_2B V0m) (ap(ap(c\_2Earithmetic\_2E\_2A \\ & V0m) V1n))))))))))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
 & (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n) = (ap (ap c\_2Earithmetic\_2E\_2A \\
 & V1n) V0m)))) \\
 \end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in ty\_2Enum\_2Enum. (\forall V1q \in ty\_2Enum\_2Enum. ( \\
 & \forall V2n \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2EXP V2n) \\
 & (ap (ap c\_2Earithmetic\_2E\_2B V0p) V1q)) = (ap (ap c\_2Earithmetic\_2E\_2A \\
 & (ap (ap c\_2Earithmetic\_2EXP V2n) V0p)) (ap (ap c\_2Earithmetic\_2EXP \\
 & V2n) V1q))))))) \\
 \end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
 & \forall V2p \in ty\_2Enum\_2Enum. (((ap (ap c\_2Earithmetic\_2E\_2A V1n) \\
 & V0m) = (ap (ap c\_2Earithmetic\_2E\_2A V2p) V0m)) \Leftrightarrow ((V0m = c\_2Enum\_2E0) \vee \\
 & (V1n = V2p))))))) \\
 \end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty\_2Enum\_2Enum. (((ap (ap c\_2Earithmetic\_2EXP \\
 & (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) \\
 & V0n) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
 & c\_2Earithmetic\_2EZERO))) \wedge ((ap (ap c\_2Earithmetic\_2EXP V0n) \\
 & (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = \\
 & V0n))) \\
 \end{aligned} \tag{15}$$

Assume the following.

$$True \tag{16}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\
 & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\
 & (p V0t)) \Leftrightarrow (p V0t)))))) \\
 \end{aligned} \tag{17}$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \tag{18}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1a \in ty\_2Enum\_2Enum. \\
 & (ap (ap c\_2Enumeral\_2Etexp\_help V0n) V1a) = (ap (ap c\_2Earithmetic\_2E\_2A \\
 & (ap (ap c\_2Earithmetic\_2E\_2B V1a) (ap c\_2Earithmetic\_2ENUMERAL \\
 & (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) (ap (ap \\
 & c\_2Earithmetic\_2EXP (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 \\
 & c\_2Earithmetic\_2EZERO))) (ap (ap c\_2Earithmetic\_2E\_2B V0n) ( \\
 & ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))))))) \\
 & (19)
 \end{aligned}$$

### Theorem 1

$$\begin{aligned}
 & (\forall V0n \in ty\_2Enum\_2Enum. ((ap (ap c\_2Enumeral\_2Etexp\_help \\
 & V0n) c\_2Enum\_2E0) = (ap (ap c\_2Earithmetic\_2EXP (ap c\_2Earithmetic\_2ENUMERAL \\
 & (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) (ap (ap c\_2Earithmetic\_2E\_2B \\
 & V0n) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
 & c\_2Earithmetic\_2EZERO)))))))
 \end{aligned}$$