

thm_2Enumeral__bit_2EBIT__MODIFY__EVAL (TMMEaH2D1rezfSG1Z32EbeA82oj6uT2sc39)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2EFUNPOW : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Earithmetic_2EFUNPOW\ A.27a \in ((A.27a^{A.27a})^{ty_2Enum_2Enum})^{(A.27a^{A.27a})} \tag{2}$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{5}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{6}$$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A.27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A.27a}))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Definition 7 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B$

Definition 8 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (8)$$

Definition 9 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 10 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V0t2))$

Definition 12 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if\ (\exists x \in A.p\ (ap\ P\ x))\ then\ (the\ (\lambda x.x \in A.\lambda y.y \in A))$ of type $\iota \Rightarrow \iota$.

Definition 13 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t2 \in 2.V0t2))$

Definition 14 We define c_2Ebit_2ESBIT to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V0t2))$

Let $c_2Ebit_2EBIT_MODIFY : \iota$ be given. Assume the following.

$$c_2Ebit_2EBIT_MODIFY \in (((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{(2^2)^{ty_2Enum_2Enum}})^{ty_2Enum_2Enum}) \quad (9)$$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (10)$$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (11)$$

Definition 15 We define $c_2Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.V0x$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (12)$$

Definition 16 We define $c_2Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.V0x$

Definition 17 We define c_2Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V$

Definition 18 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic$

Definition 19 We define c_2Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Definition 20 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (13)$$

Definition 21 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E$

Definition 22 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 23 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 24 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 25 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 26 We define $c_2Enumeral_2EiiSUC$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap c_2Enum_2ESUC (ap$

Definition 27 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 28 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 29 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap (ap (ap (c_2Ebool_2E$

Let $c_2Enumeral_2Eexp_help : \iota$ be given. Assume the following.

$$c_2Enumeral_2Eexp_help \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (14)$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (15)$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (16)$$

Let $c_2Enumeral_bit_2EBIT_MODF : \iota$ be given. Assume the following.

$$c_2Enumeral_bit_2EBIT_MODF \in ((((((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (17)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (18)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (19)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (20)$$

Definition 30 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2E$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (21)$$

Definition 31 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c^{A_27a}$

Assume the following.

$$\begin{aligned} ((\forall V0m \in ty_2Enum_2Enum. ((ap\ (ap\ c_2Earithmetic_2EEXP \\ V0m)\ c_2Enum_2E0) = (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1 \\ c_2Earithmetic_2EZERO)))) \wedge (\forall V1m \in ty_2Enum_2Enum. (\forall V2n \in \\ ty_2Enum_2Enum. ((ap\ (ap\ c_2Earithmetic_2EEXP\ V1m)\ (ap\ c_2Enum_2ESUC \\ V2n)) = (ap\ (ap\ c_2Earithmetic_2E_2A\ V1m)\ (ap\ (ap\ c_2Earithmetic_2EEXP \\ V1m)\ V2n)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0f \in (A_27a^{A_27a}). (\forall V1x \in \\ A_27a. ((ap\ (ap\ (ap\ (c_2Earithmetic_2EFUNPOW\ A_27a)\ V0f)\ c_2Enum_2E0) \\ V1x) = V1x))) \wedge (\forall V2f \in (A_27a^{A_27a}). (\forall V3n \in ty_2Enum_2Enum. \\ (\forall V4x \in A_27a. ((ap\ (ap\ (ap\ (c_2Earithmetic_2EFUNPOW\ A_27a) \\ V2f)\ (ap\ c_2Enum_2ESUC\ V3n))\ V4x) = (ap\ (ap\ (ap\ (c_2Earithmetic_2EFUNPOW \\ A_27a)\ V2f)\ V3n)\ (ap\ V2f\ V4x))))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} ((ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)) = \\ (ap\ c_2Enum_2ESUC\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1 \\ c_2Earithmetic_2EZERO)))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty_2Enum_2Enum. ((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m) \\ c_2Enum_2E0) = V0m)) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge (((ap (\\
& \quad ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B \\
& \quad V0m) V1n))) \wedge ((ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Enum_2ESUC \\
& \quad V1n)) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n))))))))) \\
& \hspace{15em} (26)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B \\
& \quad V1n) V0m)))) \\
& \hspace{15em} (27)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B \\
& \quad V1n) V0m)))) \\
& \hspace{15em} (28)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V0m) \\
& \quad (ap (ap c_2Earithmetic_2E_2B V1n) V2p)) = (ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p)))))) \\
& \hspace{15em} (29)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A V0m) \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = \\
& \quad V0m)) \\
& \hspace{15em} (30)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\
& \quad (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\
& \quad (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\
& \quad (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge (\\
& \quad ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap \\
& \quad (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\
& \quad V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = \\
& \quad (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A \\
& \quad V0m) V1n))))))))) \\
& \hspace{15em} (31)
\end{aligned}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2E_2A V0m) V1n) = (ap (ap c_2Earithmetic_2E_2A V1n) V0m)))) \quad (32)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2E_2A V0m) V1n) = (ap (ap c_2Earithmetic_2E_2A V1n) V0m)))) \quad (33)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A V0m) V2p)) = (ap (ap c_2Earithmetic_2E_2A (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) V2p)))))) \quad (34)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V0m) V2p)) = (ap (ap c_2Earithmetic_2E_2B V1n) V2p)) \Leftrightarrow (V0m = V1n)))) \quad (35)$$

Assume the following.

$$p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap (ap c_2Earithmetic_2EEXP (ap c_2Enum_2ESUC V1n)) V0m)))) \quad (36)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p)))))) \quad (37)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\neg (V0m = V1n)) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V0m)) V1n)) \vee (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V1n)) V0m)))))) \quad (38)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0n) = (ap (ap c_2Earithmetic_2E_2B (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)) V0n)))) \quad (39)$$

Assume the following.

$$(\forall V0q \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EDIV V0q) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) = V0q)) \quad (40)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V0m)) \wedge (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V1n)))) \Rightarrow (\forall V2x \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EDIV (ap (ap c_2Earithmetic_2EDIV V2x) V0m)) V1n) = (ap (ap c_2Earithmetic_2EDIV V2x) (ap (ap c_2Earithmetic_2E_2A V0m) V1n))))))) \quad (41)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0n) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) \wedge ((ap (ap c_2Earithmetic_2EEXP V0n) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0n))) \quad (42)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0f \in (A_27a^{A-27a}).(\forall V1n \in ty_2Enum_2Enum.(\forall V2x \in A_27a.((ap (ap (ap (c_2Earithmetic_2EFUNPOW A_27a) V0f) (ap c_2Enum_2ESUC V1n)) V2x) = (ap V0f (ap (ap (ap (c_2Earithmetic_2EFUNPOW A_27a) V0f) V1n) V2x))))))) \quad (43)$$

Assume the following.

$$((\forall V0f \in ((2^2)^{ty_2Enum_2Enum}).(\forall V1x \in ty_2Enum_2Enum.((ap (ap (ap c_2Ebit_2EBIT_MODIFY c_2Enum_2E0) V0f) V1x) = c_2Enum_2E0))) \wedge (\forall V2n \in ty_2Enum_2Enum.(\forall V3f \in ((2^2)^{ty_2Enum_2Enum}).(\forall V4x \in ty_2Enum_2Enum.((ap (ap (ap c_2Ebit_2EBIT_MODIFY (ap c_2Enum_2ESUC V2n)) V3f) V4x) = (ap (ap c_2Earithmetic_2E_2B (ap (ap (ap c_2Ebit_2EBIT_MODIFY V2n) V3f) V4x)) (ap (ap c_2Ebit_2ESBIT (ap (ap V3f V2n) (ap (ap c_2Ebit_2EBIT V2n) V4x))) V2n))))))) \quad (44)$$

Assume the following.

$$(\forall V0a \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D (ap c_2Enum_2ESUC V0a)) V0a) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \quad (45)$$

Assume the following.

$$\begin{aligned}
& (\forall V0h \in ty_2Enum_2Enum. (\forall V1l \in ty_2Enum_2Enum. (\\
& \quad \forall V2n \in ty_2Enum_2Enum. ((ap (ap (ap c_2Ebit_2EBITS V0h) V1l) \\
V2n) = (ap (ap c_2Earithmetic_2EMOD (ap (ap c_2Earithmetic_2EDIV \\
& \quad V2n) (ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) V1l))) (\\
& \quad ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL (ap \\
& \quad c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) (ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Enum_2ESUC V0h) V1l))))))))) \\
\end{aligned} \tag{46}$$

Assume the following.

$$((ap c_2Ebit_2EBIT c_2Enum_2E0) = c_2Earithmetic_2EODD) \tag{47}$$

Assume the following.

$$True \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
& \quad V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \\
\end{aligned} \tag{49}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \tag{50}$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \tag{51}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a. nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\
& \quad A_27a. (p V0t)) \Leftrightarrow (p V0t))) \\
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
& \quad (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \\
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\
& \quad (p V0t)) \Leftrightarrow (p V0t)))))) \\
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
& \quad ((\neg False) \Leftrightarrow True)))) \\
\end{aligned} \tag{55}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (56)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (57)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (58)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V0t1)\ V1t2) = V1t2)))) \quad (59)$$

Assume the following.

$$(\forall V0t \in 2. (((p\ V0t) \Rightarrow False) \Leftrightarrow ((p\ V0t) \Leftrightarrow False))) \quad (60)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (61)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. (\forall V5y_27 \in A_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27)\ V5y_27)))))) \quad (62)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1)\ V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V2t1)\ V3t2) = V3t2)))) \quad (63)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0f \in ((A_27a^{A_27a})^{A_27a}). \\
& \quad ((\forall V1x \in A_27a. (\forall V2y \in A_27a. (\forall V3z \in A_27a. \\
& ((ap\ (ap\ V0f\ V1x)\ (ap\ (ap\ V0f\ V2y)\ V3z)) = (ap\ (ap\ V0f\ (ap\ (ap\ V0f\ V1x)\ \\
& \quad V2y))\ V3z)))) \Rightarrow ((\forall V4x \in A_27a. (\forall V5y \in A_27a. ((ap\ \\
& (ap\ V0f\ V4x)\ V5y) = (ap\ (ap\ V0f\ V5y)\ V4x)))) \Rightarrow (\forall V6x \in A_27a. (\\
& \quad \forall V7y \in A_27a. (\forall V8z \in A_27a. ((ap\ (ap\ V0f\ V6x)\ (ap\ (ap\ \\
& V0f\ V7y)\ V8z)) = (ap\ (ap\ V0f\ V7y)\ (ap\ (ap\ V0f\ V6x)\ V8z))))))))) \\
& \hspace{15em} (64)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty_2Enum_2Enum}). ((p\ (ap\ V0P\ c_2Enum_2E0)) \wedge \\
& (\forall V1n \in ty_2Enum_2Enum. ((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c_2Enum_2ESUC \\
& \quad V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum. (p\ (ap\ V0P\ V2n)))))) \\
& \hspace{15em} (65)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (((ap\ c_2Enum_2ESUC\ c_2Earithmetic_2EZERO) = (ap\ c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO)) \wedge ((\forall V0n \in ty_2Enum_2Enum. ((ap\ \\
& c_2Enum_2ESUC\ (ap\ c_2Earithmetic_2EBIT1\ V0n)) = (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))) \wedge (\forall V1n \in ty_2Enum_2Enum. ((ap\ c_2Enum_2ESUC\ (ap\ c_2Earithmetic_2EBIT2 \\
& \quad V1n)) = (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enum_2ESUC\ V1n)))))) \\
& \hspace{15em} (66)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (\\
& ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (\\
& ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (\\
& ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap \\
& c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC \\
& (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ c_2Earithmetic_2EZERO)) = (\\
& ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (\\
& ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT1 \\
& V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m))))))))))))))))))))))))))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmic_2EZERO) (ap c_2Earithmic_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmic_2EZERO) \\
& (ap c_2Earithmic_2EBIT2 V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& V0n) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT1 V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT2 V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT1 V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow \\
& (\neg(p (ap (ap c_2Eprim_rec_2E_3C V1m) V0n))) \wedge ((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT2 V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))))))))))
\end{aligned} \tag{69}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Earithmic_2E_3C_3D c_2Earithmic_2EZERO) V0n))) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D \\
& (ap c_2Earithmic_2EBIT2 V0n)) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT2 \\
& V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow (\neg(p (ap (ap c_2Earithmic_2E_3C_3D \\
& V1m) V0n)))) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT2 \\
& V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))))))))))
\end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned}
& (((ap c_2Eprim_rec_2EPRE c_2Earithmic_2EZERO) = c_2Earithmic_2EZERO) \wedge \\
& (((ap c_2Eprim_rec_2EPRE (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)) = \\
& c_2Earithmic_2EZERO) \wedge ((\forall V0n \in ty_2Enum_2Enum. ((ap \\
& c_2Eprim_rec_2EPRE (ap c_2Earithmic_2EBIT1 (ap c_2Earithmic_2EBIT1 \\
& V0n))) = (ap c_2Earithmic_2EBIT2 (ap c_2Eprim_rec_2EPRE (ap \\
& c_2Earithmic_2EBIT1 V0n)))))) \wedge ((\forall V1n \in ty_2Enum_2Enum. \\
& ((ap c_2Eprim_rec_2EPRE (ap c_2Earithmic_2EBIT1 (ap c_2Earithmic_2EBIT2 \\
& V1n))) = (ap c_2Earithmic_2EBIT2 (ap c_2Earithmic_2EBIT1 \\
& V1n)))))) \wedge ((\forall V2n \in ty_2Enum_2Enum. ((ap c_2Eprim_rec_2EPRE \\
& (ap c_2Earithmic_2EBIT2 V2n)) = (ap c_2Earithmic_2EBIT1 V2n))))))
\end{aligned} \tag{71}$$

Assume the following.

$$\begin{aligned}
& (\forall V0acc \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
& (((ap (ap c_2Enumeral_2Etxp_help c_2Earithmetic_2EZERO) V0acc) = \\
& (ap c_2Earithmetic_2EBIT2 V0acc)) \wedge (((ap (ap c_2Enumeral_2Etxp_help \\
& (ap c_2Earithmetic_2EBIT1 V1n)) V0acc) = (ap (ap c_2Enumeral_2Etxp_help \\
& (ap c_2Eprim_rec_2EPRE (ap c_2Earithmetic_2EBIT1 V1n)) (ap \\
& c_2Earithmetic_2EBIT1 V0acc))) \wedge ((ap (ap c_2Enumeral_2Etxp_help \\
& (ap c_2Earithmetic_2EBIT2 V1n)) V0acc) = (ap (ap c_2Enumeral_2Etxp_help \\
& (ap c_2Earithmetic_2EBIT1 V1n)) (ap c_2Earithmetic_2EBIT1 V0acc))))))))) \\
& \tag{72}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (((ap (ap c_2Earithmetic_2EEXP \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) \\
& c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO))) \wedge (((ap (ap c_2Earithmetic_2EEXP (ap \\
& c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V0n))) = \\
& (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Enumeral_2Etxp_help \\
& (ap c_2Eprim_rec_2EPRE (ap c_2Earithmetic_2EBIT1 V0n))) c_2Earithmetic_2EZERO))) \wedge \\
& ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL (\\
& ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT2 V0n))) = (ap c_2Earithmetic_2ENUMERAL \\
& (ap (ap c_2Enumeral_2Etxp_help (ap c_2Earithmetic_2EBIT1 V0n)) \\
& c_2Earithmetic_2EZERO))))))))) \\
& \tag{73}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0f \in ((2^2)^{ty_2Enum_2Enum}). (\forall V1x \in ty_2Enum_2Enum. \\
& (\forall V2b \in ty_2Enum_2Enum. (\forall V3e \in ty_2Enum_2Enum. (\\
& \forall V4y \in ty_2Enum_2Enum. ((ap (ap (ap (ap (ap (ap c_2Enumeral_bit_2EBIT_MODF \\
& c_2Enum_2E0) V0f) V1x) V2b) V3e) V4y) = V4y)))))) \wedge (\forall V5n \in \\
& ty_2Enum_2Enum. (\forall V6f \in ((2^2)^{ty_2Enum_2Enum}). (\forall V7x \in \\
& ty_2Enum_2Enum. (\forall V8b \in ty_2Enum_2Enum. (\forall V9e \in ty_2Enum_2Enum. \\
& (\forall V10y \in ty_2Enum_2Enum. ((ap (ap (ap (ap (ap (ap c_2Enumeral_bit_2EBIT_MODF \\
& (ap c_2Enum_2ESUC V5n)) V6f) V7x) V8b) V9e) V10y) = (ap (ap (ap (ap \\
& (ap (ap c_2Enumeral_bit_2EBIT_MODF V5n) V6f) (ap (ap c_2Earithmetic_2EDIV \\
& V7x) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 \\
& c_2Earithmetic_2EZERO)))) (ap (ap c_2Earithmetic_2E_2B V8b) \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \\
& (ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL (ap \\
& c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) V9e)) (ap (ap \\
& (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) (ap (ap V6f V8b) (ap c_2Earithmetic_2EODD \\
& V7x))) (ap (ap c_2Earithmetic_2E_2B V9e) V10y)) V10y))))))))) \\
& \tag{74}
\end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\ & \quad A_27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\ & \quad (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \\ & \hspace{15em} (75) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0x \in A_27a. (\forall V1y \in A_27b. ((ap\ (c_2Epair_2EFST\ A_27a \\ & \quad A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y)) = V0x))) \\ & \hspace{15em} (76) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & \quad nonempty\ A_27c \Rightarrow (\forall V0f \in ((A_27c^{A_27b})^{A_27a}). (\forall V1x \in \\ & \quad A_27a. (\forall V2y \in A_27b. ((ap\ (ap\ (c_2Epair_2EUNCURRY\ A_27a \\ & \quad A_27b\ A_27c)\ V0f)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V1x)\ V2y)) = \\ & \quad (ap\ (ap\ V0f\ V1x)\ V2y)))))) \\ & \hspace{15em} (77) \end{aligned}$$

Theorem 1

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1f \in ((2^2)^{ty_2Enum_2Enum}), \\ & \quad (\forall V2n \in ty_2Enum_2Enum. ((ap\ (ap\ (ap\ c_2Ebit_2EBIT_MODIFY \\ & \quad V0m)\ V1f)\ V2n) = (ap\ (ap\ (ap\ (ap\ (ap\ (ap\ c_2Enumeral_bit_2EBIT_MODF \\ & \quad V0m)\ V1f)\ V2n)\ c_2Enum_2E0)\ (ap\ c_2Earithmic_2ENUMERAL\ (ap\ c_2Earithmic_2EBIT1 \\ & \quad c_2Earithmic_2EZERO)))\ c_2Enum_2E0)))))) \end{aligned}$$