

# thm\_2Enumeral\_\_bit\_2EBIT\_\_REVERSE\_\_EVAL (TMLENSybuQUaSFPp3b4mc8Zj767Jd2yEjF)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $c\_2Earithmetic\_2EFUNPOW : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.\ 27a.nonempty\ A.\ 27a \Rightarrow c\_2Earithmetic\_2EFUNPOW\ A.\ 27a \in ((A.\ 27a^{A.\ 27a})^{ty\_2Enum\_2Enum})^{(A.\ 27a^{A.\ 27a})} \tag{2}$$

Let  $c\_2Ebit\_2EBIT\_REVERSE : \iota$  be given. Assume the following.

$$c\_2Ebit\_2EBIT\_REVERSE \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \tag{3}$$

Let  $c\_2Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \tag{4}$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{5}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum)^{\omega} \tag{6}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\ \lambda x \in A.\ \lambda y \in A.\ inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 3** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega)^{ty\_2Enum\_2Enum} \tag{7}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega)^{\omega} \tag{8}$$

**Definition 4** We define  $c\_Ebool\_2ET$  to be  $(ap (ap (c\_Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_Ebool\_2E\_21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_Emin\_2E\_3D (2^{A-27a})) (\lambda V1x \in 2.V1x)) (\lambda V2t \in 2.V2t)))$

**Definition 6** We define  $c\_Eenum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_Eenum\_2EABS\_num (ap (ap (c\_Emin\_2E\_3D (2^2)) (\lambda V1x \in 2.V1x)) (\lambda V2t \in 2.V2t)))$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (9)$$

**Definition 7** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E\_2B (ap (ap (c\_Emin\_2E\_3D (2^2)) (\lambda V1x \in 2.V1x)) (\lambda V2t \in 2.V2t))))$

**Definition 8** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Earithmetic\_2EEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (10)$$

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (11)$$

**Definition 9** We define  $c\_2Ebit\_2EDIV\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2EDIV (ap (ap c\_2Earithmetic\_2EEXP (ap (ap (c\_Emin\_2E\_3D (2^2)) (\lambda V1x \in 2.V1x)) (\lambda V2t \in 2.V2t))))))$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (12)$$

**Definition 10** We define  $c\_2Ebit\_2EMOD\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2EDIV (ap (ap c\_2Earithmetic\_2EEXP (ap (ap (c\_Emin\_2E\_3D (2^2)) (\lambda V1x \in 2.V1x)) (\lambda V2t \in 2.V2t))))))$

**Definition 11** We define  $c\_2Ebit\_2EBITS$  to be  $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V2t \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2EDIV (ap (ap c\_2Earithmetic\_2EEXP (ap (ap (c\_Emin\_2E\_3D (2^2)) (\lambda V1x \in 2.V1x)) (\lambda V2t \in 2.V2t))))))$

**Definition 12** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2EBIT2 (ap (ap (c\_Emin\_2E\_3D (2^2)) (\lambda V1x \in 2.V1x)) (\lambda V2t \in 2.V2t))))$

**Definition 13** We define  $c\_2Ebit\_2EBIT$  to be  $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2EBIT1 (ap (ap (c\_Emin\_2E\_3D (2^2)) (\lambda V1x \in 2.V1x)) (\lambda V2t \in 2.V2t))))$

**Definition 14** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (13)$$

**Definition 15** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 16** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_7E))$

**Definition 17** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))))$

**Definition 18** We define  $c\_Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.$ **if**  $(\exists x \in A.p (ap P x))$  **then**  $(the (\lambda x.x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 19** We define  $c\_Ebool\_2E\_3F$  to be  $\lambda A.\lambda P \in 2^A.(ap V0P (ap (c\_Emin\_2E\_40 P)))$

**Definition 20** We define  $c\_Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 21** We define  $c\_Earithmic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 22** We define  $c\_Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_2E\_21 2) (\lambda V2t \in 2.))$

**Definition 23** We define  $c\_Earithmic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 24** We define  $c\_Earithmic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 25** We define  $c\_Eenumerat\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 26** We define  $c\_Ebool\_2ECOND$  to be  $\lambda A.\lambda P \in 2^A.(ap (\lambda V0t \in 2.(\lambda V1t1 \in A.\lambda V2t2 \in A.))$

**Definition 27** We define  $c\_Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap (ap (ap (c\_Ebool\_2ECOND$

Let  $c\_Eenumerat\_2Eexp\_help : \iota$  be given. Assume the following.

$$c\_Eenumerat\_2Eexp\_help \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (14)$$

Let  $c\_Earithmic\_2EODD : \iota$  be given. Assume the following.

$$c\_Earithmic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (15)$$

**Definition 28** We define  $c\_Ebit\_2ESBIT$  to be  $\lambda V0b \in 2.\lambda V1n \in ty\_2Enum\_2Enum.(ap (ap (ap (c\_Ebool\_2ECOND$

Let  $c\_Earithmic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_Earithmic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (16)$$

Let  $c\_Eenumerat\_bit\_2EBIT\_REV : \iota$  be given. Assume the following.

$$c\_Eenumerat\_bit\_2EBIT\_REV \in (((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (17)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (18)$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.\lambda A\_27a \in A.\lambda A\_27b \in A.(ap (ap (ap (c\_2Epair\_2ESND A\_27a A\_27b) (ty\_2Epair\_2Eprod A\_27a A\_27b))) \quad (19)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.\lambda A\_27a \in A.\lambda A\_27b \in A.(ap (ap (ap (c\_2Epair\_2EABS\_prod A\_27a A\_27b) (ty\_2Epair\_2Eprod A\_27a A\_27b))^{(2^{A\_27b})^{A\_27a}})) \quad (20)$$

**Definition 29** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota)$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EFST \\ A\_27a A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod A\_27a A\_27b)}) \quad (21)$$

**Definition 30** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27a})$   
Assume the following.

$$((\forall V0m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\ V0m) c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\ c\_2Earithmetic\_2EZERO)))) \wedge (\forall V1m \in ty\_2Enum\_2Enum.(\forall V2n \in \\ ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V1m) (ap c\_2Enum\_2ESUC \\ V2n)) = (ap (ap c\_2Earithmetic\_2E\_2A V1m) (ap (ap c\_2Earithmetic\_2EEXP \\ V1m) V2n))))))) \quad (22)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow ((\forall V0f \in (A\_27a^{A\_27a}).(\forall V1x \in \\ A\_27a.((ap (ap (ap (c\_2Earithmetic\_2EFUNPOW A\_27a) V0f) c\_2Enum\_2E0) \\ V1x) = V1x))) \wedge (\forall V2f \in (A\_27a^{A\_27a}).(\forall V3n \in ty\_2Enum\_2Enum. \\ (\forall V4x \in A\_27a.((ap (ap (ap (c\_2Earithmetic\_2EFUNPOW A\_27a) \\ V2f) (ap c\_2Enum\_2ESUC V3n)) V4x) = (ap (ap (ap (c\_2Earithmetic\_2EFUNPOW \\ A\_27a) V2f) V3n) (ap V2f V4x))))))) \quad (23)$$

Assume the following.

$$((ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)) = \\ (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\ c\_2Earithmetic\_2EZERO)))) \quad (24)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n) = (ap (ap c\_2Earithmetic\_2E\_2A \\ V1n) V0m)))) \quad (25)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ \forall V2p \in ty\_2Enum\_2Enum.(((ap (ap c\_2Earithmetic\_2E\_2B V0m) \\ V2p) = (ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p)) \Leftrightarrow (V0m = V1n)))) \quad (26)$$

Assume the following.

$$p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap (ap c\_2Earithmetic\_2EEXP \\ (ap c\_2Enum\_2ESUC V1n) V0m)))) \quad (27)$$

Assume the following.

$$\begin{aligned}
& (\forall V0q \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2EDIV V0q) \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = \\
& \quad V0q))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) V0m)) \wedge (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& c\_2Enum\_2E0) V1n))) \Rightarrow (\forall V2x \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2EDIV \\
& (ap (ap c\_2Earithmetic\_2EDIV V2x) V0m)) V1n) = (ap (ap c\_2Earithmetic\_2EDIV \\
& V2x) (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (((ap (ap c\_2Earithmetic\_2EEXP \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) \\
& V0n) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& c\_2Earithmetic\_2EZERO))) \wedge ((ap (ap c\_2Earithmetic\_2EEXP V0n) \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = \\
& \quad V0n)))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0f \in (A\_27a^{A\_27a}). (\forall V1n \in \\
& ty\_2Enum\_2Enum. (\forall V2x \in A\_27a. ((ap (ap (ap (c\_2Earithmetic\_2EFUNPOW \\
& A\_27a) V0f) (ap c\_2Enum\_2ESUC V1n)) V2x) = (ap V0f (ap (ap (c\_2Earithmetic\_2EFUNPOW \\
& A\_27a) V0f) V1n) V2x))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0x \in ty\_2Enum\_2Enum. ((ap (ap c\_2Ebit\_2EBIT\_REVERSE \\
& c\_2Enum\_2E0) V0x) = c\_2Enum\_2E0)) \wedge (\forall V1n \in ty\_2Enum\_2Enum. \\
& (\forall V2x \in ty\_2Enum\_2Enum. ((ap (ap c\_2Ebit\_2EBIT\_REVERSE \\
& (ap c\_2Enum\_2ESUC V1n)) V2x) = (ap (ap c\_2Earithmetic\_2E\_2B (ap \\
& (ap c\_2Earithmetic\_2E\_2A (ap (ap c\_2Ebit\_2EBIT\_REVERSE V1n) \\
& V2x)) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 \\
& c\_2Earithmetic\_2EZERO)))) (ap (ap c\_2Ebit\_2ESBIT (ap (ap c\_2Ebit\_2EBIT \\
& V1n) V2x)) c\_2Enum\_2E0))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2D ( \\
& ap c\_2Enum\_2ESUC V0a)) V0a) = (ap c\_2Earithmetic\_2ENUMERAL (ap \\
& c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0h \in ty\_2Enum\_2Enum. (\forall V1l \in ty\_2Enum\_2Enum. ( \\
& \quad \forall V2n \in ty\_2Enum\_2Enum. ((ap (ap (ap c\_2Ebit\_2EBITS V0h) V1l) \\
V2n) = (ap (ap c\_2Earithmetic\_2EMOD (ap (ap c\_2Earithmetic\_2EDIV \\
& \quad V2n) (ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad \quad (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)))) V1l))) ( \\
& \quad \quad ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL (ap \\
& \quad \quad c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) (ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad \quad \quad (ap c\_2Enum\_2ESUC V0h) V1l)))))))))
\end{aligned} \tag{34}$$

Assume the following.

$$((ap c\_2Ebit\_2EBIT c\_2Enum\_2E0) = c\_2Earithmetic\_2EODD) \tag{35}$$

Assume the following.

$$True \tag{36}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\
& \quad A\_27a.(p V0t)) \Leftrightarrow (p V0t)))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
& \quad (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow \\
& \quad True))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in \\
& \quad A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\
& \quad p V0t))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\
& ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p (ap V0P c\_2Enum\_2E0)) \wedge \\ & (\forall V1n \in ty\_2Enum\_2Enum.((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c\_2Enum\_2ESUC \\ & V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p (ap V0P V2n)))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enumeral\_2EiZ (ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\
& \quad ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A ( \\
& \quad ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\
& \quad c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\
& \quad ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V14n) c\_2Enum\_2E0) = \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap (ap c\_2Earithmetic\_2EEXP V15n) V16m)))))) \wedge ((ap c\_2Enum\_2ESUC \\
& \quad c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum.( \\
& \quad (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Enum\_2ESUC V17n)))) \wedge ((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\
& \quad c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Eprim\_rec\_2EPRE V18n)))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\
& \quad (((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E \\
& \quad c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL
\end{aligned}$$



Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) (ap c\_2Earithmetic\_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& (ap c\_2Earithmetic\_2EBIT2 V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& V0n) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmetic\_2EBIT1 V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) V1m))) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmetic\_2EBIT2 V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) V1m))) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmetic\_2EBIT1 V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow \\
& (\neg (p (ap (ap c\_2Eprim\_rec\_2E\_3C V1m) V0n))) \wedge ((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmetic\_2EBIT2 V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) V1m))))))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (((ap c\_2Eprim\_rec\_2EPRE c\_2Earithmetic\_2EZERO) = c\_2Earithmetic\_2EZERO) \wedge \\
& (((ap c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)) = \\
& c\_2Earithmetic\_2EZERO) \wedge ((\forall V0n \in ty\_2Enum\_2Enum. ((ap \\
& c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmetic\_2EBIT1 (ap c\_2Earithmetic\_2EBIT1 \\
& V0n))) = (ap c\_2Earithmetic\_2EBIT2 (ap c\_2Eprim\_rec\_2EPRE (ap \\
& c\_2Earithmetic\_2EBIT1 V0n)))))) \wedge ((\forall V1n \in ty\_2Enum\_2Enum. \\
& ((ap c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmetic\_2EBIT1 (ap c\_2Earithmetic\_2EBIT2 \\
& V1n))) = (ap c\_2Earithmetic\_2EBIT2 (ap c\_2Earithmetic\_2EBIT1 \\
& V1n)))) \wedge ((\forall V2n \in ty\_2Enum\_2Enum. ((ap c\_2Eprim\_rec\_2EPRE \\
& (ap c\_2Earithmetic\_2EBIT2 V2n)) = (ap c\_2Earithmetic\_2EBIT1 V2n))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0acc \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
& (((ap (ap c\_2Enumeral\_2Etexp\_help c\_2Earithmetic\_2EZERO) V0acc) = \\
& (ap c\_2Earithmetic\_2EBIT2 V0acc)) \wedge (((ap (ap c\_2Enumeral\_2Etexp\_help \\
& (ap c\_2Earithmetic\_2EBIT1 V1n)) V0acc) = (ap (ap c\_2Enumeral\_2Etexp\_help \\
& (ap c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmetic\_2EBIT1 V1n)) (ap \\
& c\_2Earithmetic\_2EBIT1 V0acc))) \wedge ((ap (ap c\_2Enumeral\_2Etexp\_help \\
& (ap c\_2Earithmetic\_2EBIT2 V1n)) V0acc) = (ap (ap c\_2Enumeral\_2Etexp\_help \\
& (ap c\_2Earithmetic\_2EBIT1 V1n)) (ap c\_2Earithmetic\_2EBIT1 V0acc))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (((ap (ap c\_2Earithmetic\_2EEXP \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) \\
& c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& c\_2Earithmetic\_2EZERO))) \wedge (((ap (ap c\_2Earithmetic\_2EEXP (ap \\
& c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 V0n))) = \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Enumeral\_2Eexp\_help \\
& (ap c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmetic\_2EBIT1 V0n))) c\_2Earithmetic\_2EZERO))) \wedge \\
& (((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL ( \\
& ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT2 V0n))) = (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap (ap c\_2Enumeral\_2Eexp\_help (ap c\_2Earithmetic\_2EBIT1 V0n)) \\
& c\_2Earithmetic\_2EZERO))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0x \in ty\_2Enum\_2Enum. (\forall V1y \in ty\_2Enum\_2Enum. \\
& ((ap (ap (ap c\_2Enumeral\_bit\_2EBIT\_REV c\_2Enum\_2E0) V0x) V1y) = \\
& V1y))) \wedge (\forall V2n \in ty\_2Enum\_2Enum. (\forall V3x \in ty\_2Enum\_2Enum. \\
& (\forall V4y \in ty\_2Enum\_2Enum. ((ap (ap (ap c\_2Enumeral\_bit\_2EBIT\_REV \\
& (ap c\_2Enum\_2ESUC V2n)) V3x) V4y) = (ap (ap (ap c\_2Enumeral\_bit\_2EBIT\_REV \\
& V2n) (ap (ap c\_2Earithmetic\_2EDIV V3x) (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)))) (ap (ap \\
& c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) V4y)) (ap \\
& (ap c\_2Ebit\_2ESBIT (ap c\_2Earithmetic\_2EODD V3x)) c\_2Enum\_2E0)))))))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( \\
& \forall V0x \in A\_27a. (\forall V1y \in A\_27b. (\forall V2a \in A\_27a. (\forall V3b \in \\
& A\_27b. (((ap (ap (c\_2Epair\_2E\_2C A\_27a A\_27b) V0x) V1y) = (ap (ap \\
& (c\_2Epair\_2E\_2C A\_27a A\_27b) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( \\
& \forall V0x \in A\_27a. (\forall V1y \in A\_27b. ((ap (c\_2Epair\_2ESND A\_27a \\
& A\_27b) (ap (ap (c\_2Epair\_2E\_2C A\_27a A\_27b) V0x) V1y)) = V1y)))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
& \quad nonempty\ A\_27c \Rightarrow (\forall V0f \in ((A\_27c^{A\_27b})^{A\_27a}). (\forall V1x \in \\
& \quad A\_27a. (\forall V2y \in A\_27b. ((ap\ (ap\ (c\_2Epair\_2EUNCURRY\ A\_27a \\
& \quad A\_27b\ A\_27c)\ V0f)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V1x)\ V2y))) = \\
& \quad (ap\ (ap\ V0f\ V1x)\ V2y))))))
\end{aligned} \tag{52}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (ap\ (ap\ c\_2Ebit\_2EBIT\_REVERSE\ V0m)\ V1n) = (ap\ (ap\ (ap\ c\_2Enumeral\_bit\_2EBIT\_REV \\
& \quad V0m)\ V1n)\ c\_2Enum\_2E0))))
\end{aligned}$$