

# thm\_2Enumeral\_bit\_2EFDUB\_FDUB (TMN- WQJ5oNrKyVkjNQkqEgdVquZfQNKrdV8x)

October 26, 2020

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 3** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{5}$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ (ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

**Definition 7** We define  $c\_2Earithmic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmic\_2EBIT1 V0n) c\_2Enum\_2E0))$ .

**Definition 8** We define  $c\_2Earithmic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmic\_2EBIT2 V0n) c\_2Enum\_2E0))$ .

**Definition 9** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21) (\lambda V0t \in 2.V0t))$ .

**Definition 10** We define  $c\_2Enumeral\_bit\_2EiSUC$  to be  $c\_2Enum\_2ESUC$ .

Let  $c\_2Enumeral\_bit\_2EFDUB : \iota$  be given. Assume the following.

$$c\_2Enumeral\_bit\_2EFDUB \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})}) \quad (7)$$

**Definition 11** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 12** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21) (\lambda V2t \in 2.V2t) V1t2)))$ .

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ & ((ap (ap c\_2Earithmic\_2E\_2B c\_2Enum\_2E0) V0m) = V0m) \wedge (((ap ( \\ & ap c\_2Earithmic\_2E\_2B V0m) c\_2Enum\_2E0) = V0m) \wedge (((ap (ap c\_2Earithmic\_2E\_2B \\ & (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmic\_2E\_2B \\ & V0m) V1n)))) \wedge ((ap (ap c\_2Earithmic\_2E\_2B V0m) (ap c\_2Enum\_2ESUC \\ & V1n)) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmic\_2E\_2B V0m) V1n)))))))) \end{aligned} \quad (8)$$

Assume the following.

$$True \quad (9)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t) \Leftrightarrow (p V1x))) \Leftrightarrow (p V0t))) \quad (10)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (11)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (12)$$

Assume the following.

$$\begin{aligned} & ((\forall V0f \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}).((ap (ap c\_2Enumeral\_bit\_2EFDUB \\ & V0f) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge (\forall V1f \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}). \\ & (\forall V2n \in ty\_2Enum\_2Enum.((ap (ap c\_2Enumeral\_bit\_2EFDUB \\ & V1f) (ap c\_2Enum\_2ESUC V2n)) = (ap V1f (ap V1f (ap c\_2Enum\_2ESUC V2n)))))))) \end{aligned} \quad (13)$$

**Theorem 1**

$$\begin{aligned} & (\forall V0f \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}).(((ap (ap c\_2Enumeral\_bit\_2EFDUB \\ & (ap c\_2Enumeral\_bit\_2EFDUB V0f)) c\_2Earithmetic\_2EZERO) = c\_2Earithmetic\_2EZERO) \wedge \\ & ((\forall V1x \in ty\_2Enum\_2Enum.((ap (ap c\_2Enumeral\_bit\_2EFDUB \\ & (ap c\_2Enumeral\_bit\_2EFDUB V0f)) (ap c\_2Enumeral\_bit\_2EiSUC \\ & V1x)) = (ap (ap c\_2Enumeral\_bit\_2EFDUB V0f) (ap (ap c\_2Enumeral\_bit\_2EFDUB \\ & V0f) (ap c\_2Enumeral\_bit\_2EiSUC V1x)))))) \wedge ((\forall V2x \in ty\_2Enum\_2Enum. \\ & ((ap (ap c\_2Enumeral\_bit\_2EFDUB (ap c\_2Enumeral\_bit\_2EFDUB \\ & V0f)) (ap c\_2Earithmetic\_2EBIT1 V2x)) = (ap (ap c\_2Enumeral\_bit\_2EFDUB \\ & V0f) (ap (ap c\_2Enumeral\_bit\_2EFDUB V0f) (ap c\_2Earithmetic\_2EBIT1 \\ & V2x)))))) \wedge ((\forall V3x \in ty\_2Enum\_2Enum.((ap (ap c\_2Enumeral\_bit\_2EFDUB \\ & (ap c\_2Enumeral\_bit\_2EFDUB V0f)) (ap c\_2Earithmetic\_2EBIT2 \\ & V3x)) = (ap (ap c\_2Enumeral\_bit\_2EFDUB V0f) (ap (ap c\_2Enumeral\_bit\_2EFDUB \\ & V0f) (ap c\_2Earithmetic\_2EBIT2 V3x)))))))))) \end{aligned}$$