

thm_2Enumeral__bit_2ENUMERAL__BIT__MODF
 (TMRZowqGXSG-
 woZCkDm7Z1p8GgTDLJQtLZyZ)

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (c_2Enum_2ESUC_REP\ m))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 7 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B))$

Definition 8 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Definition 9 We define $c_2Earithmetic_2EDIV2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B))$

Definition 10 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x)) \text{ of type } \iota \Rightarrow \iota$.

Definition 11 We define $c_2Ebool_2E_3F$ to be $\lambda A.\lambda P \in 2^A.(ap V0P (ap (c_2Emin_2E_40)))$

Definition 12 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 13 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2)) (\lambda V2t \in 2.V0t))$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (8)$$

Definition 14 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21) 2) (\lambda V0t \in 2.V0t)$.

Definition 15 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21))$

Definition 16 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2)) (\lambda V2t \in 2.V0t))$

Definition 17 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.V0n$

Definition 18 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.V0n$

Definition 19 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.V0n$

Definition 20 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.V0n$

Definition 21 We define c_2Ebool_2ECOND to be $\lambda A.\lambda P \in 2^A.(ap V0P (ap (c_2Emin_2E_40)))$

Definition 22 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap (ap (ap (c_2Ebool_2E_21) 2) V0m) V0m))$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (9)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (10)$$

Definition 23 We define $c_2Enumeral_2EiSUC$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ c_2Enum_2ESUC\ (ap$

Definition 24 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 25 We define $c_2Enumeral_2EiDUB$ to be $\lambda V0x \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2D$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (11)$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (12)$$

Definition 26 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B$

Let $c_2Enumeral_bit_2EBIT_MODF : \iota$ be given. Assume the following.

$$c_2Enumeral_bit_2EBIT_MODF \in ((((((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (13)$$

Assume the following.

$$\begin{aligned} & ((\forall V0n \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2B \\ & \quad c_2Enum_2E0)\ V0n) = V0n)) \wedge (\forall V1m \in ty_2Enum_2Enum.(\forall V2n \in \\ & \quad ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Enum_2ESUC \\ & \quad V1m))\ V2n) = (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V1m) \\ & \quad V2n)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2A\ (\\ & \quad ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))) \\ & \quad V0m) = V0m)) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & \quad (ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ V1n) = (ap\ (ap\ c_2Earithmetic_2E_2A \\ & \quad V1n)\ V0m)))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A (ap \\
& (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p) = (ap (ap c_2Earithmetic_2E_2B \\
& (ap (ap c_2Earithmetic_2E_2A V0m) V2p)) (ap (ap c_2Earithmetic_2E_2A \\
& \quad V1n) V2p))))))
\end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1k \in ty_2Enum_2Enum. (\\
& \quad \forall V2q \in ty_2Enum_2Enum. ((\exists V3r \in ty_2Enum_2Enum. (\\
& (V1k = (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A \\
& V2q) V0n)) V3r)) \wedge (p (ap (ap c_2Eprim_rec_2E_3C V3r) V0n)))) \Rightarrow (\\
& (ap (ap c_2Earithmetic_2EDIV V1k) V0n) = V2q))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0f \in ((A_27a^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}). \\
& \quad (\forall V1g \in (A_27a^{ty_2Enum_2Enum}). ((\forall V2n \in ty_2Enum_2Enum. \\
& ((ap V1g (ap c_2Enum_2ESUC V2n)) = (ap (ap V0f V2n) (ap c_2Enum_2ESUC \\
& V2n)))) \Leftrightarrow ((\forall V3n \in ty_2Enum_2Enum. ((ap V1g (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 V3n)) = (ap (ap V0f (ap (ap c_2Earithmetic_2E_2D \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V3n))) \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V3n)))))) \wedge \\
& (\forall V4n \in ty_2Enum_2Enum. ((ap V1g (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT2 V4n)) = (ap (ap V0f (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 V4n)) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT2 V4n))))))))))
\end{aligned} \tag{19}$$

Assume the following.

$$True \tag{20}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
& V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))))
\end{aligned} \tag{21}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \tag{22}$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee \neg(p V0t))) \tag{23}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge \\
& ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (25)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))) \quad (26)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow \\
& True)) \quad (27)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\
& A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (28)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p \ V0t)))))) \quad (29)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in \\
& A_27a.(((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2ET) \ V0t1) \\
& V1t2) = V0t1) \wedge ((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2EF) \\
& V0t1) \ V1t2) = V1t2)))))) \quad (30)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\
& ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (31)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\
& (\forall V2x \in A_27a.(\forall V3x_27 \in A_27a.(\forall V4y \in A_27a. \\
& (\forall V5y_27 \in A_27a.(((p \ V0P) \Leftrightarrow (p \ V1Q)) \wedge (((p \ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\
& ((\neg(p \ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \\
& V0P) \ V2x) \ V4y) = (ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ V1Q) \ V3x_27) \\
& V5y_27)))))) \quad (32)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
 &(((ap\ c_2Enum_2ESUC\ c_2Earithmetic_2EZERO) = (ap\ c_2Earithmetic_2EBIT1 \\
 &\quad c_2Earithmetic_2EZERO)) \wedge (\forall V0n \in ty_2Enum_2Enum. ((ap \\
 &\quad c_2Enum_2ESUC\ (ap\ c_2Earithmetic_2EBIT1\ V0n)) = (ap\ c_2Earithmetic_2EBIT2 \\
 &\quad V0n))) \wedge (\forall V1n \in ty_2Enum_2Enum. ((ap\ c_2Enum_2ESUC\ (ap\ c_2Earithmetic_2EBIT2 \\
 &\quad V1n)) = (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enum_2ESUC\ V1n))))))
 \end{aligned}
 \tag{33}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (\\
& ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (\\
& ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (\\
& ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap \\
& c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC \\
& (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ c_2Earithmetic_2EZERO)) = (\\
& ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (\\
& ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT1 \\
& V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) (ap c_2Earithmetic_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& (ap c_2Earithmetic_2EBIT2 V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& V0n) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT1 V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT2 V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT1 V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow \\
& (\neg (p (ap (ap c_2Eprim_rec_2E_3C V1m) V0n))) \wedge ((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT2 V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))))))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0f \in ((2^2)^{ty_2Enum_2Enum}). (\forall V1x \in ty_2Enum_2Enum. \\
& (\forall V2b \in ty_2Enum_2Enum. (\forall V3e \in ty_2Enum_2Enum. (\\
& \forall V4y \in ty_2Enum_2Enum. ((ap (ap (ap (ap (ap (ap c_2Enumeral_bit_2EBIT_MODF \\
& c_2Enum_2E0) V0f) V1x) V2b) V3e) V4y) = V4y)))))) \wedge (\forall V5n \in \\
& ty_2Enum_2Enum. (\forall V6f \in ((2^2)^{ty_2Enum_2Enum}). (\forall V7x \in \\
& ty_2Enum_2Enum. (\forall V8b \in ty_2Enum_2Enum. (\forall V9e \in ty_2Enum_2Enum. \\
& (\forall V10y \in ty_2Enum_2Enum. ((ap (ap (ap (ap (ap (ap c_2Enumeral_bit_2EBIT_MODF \\
& (ap c_2Enum_2ESUC V5n)) V6f) V7x) V8b) V9e) V10y) = (ap (ap (ap (ap \\
& (ap (ap c_2Enumeral_bit_2EBIT_MODF V5n) V6f) (ap (ap c_2Earithmetic_2EDIV \\
& V7x) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 \\
& c_2Earithmetic_2EZERO)))) (ap (ap c_2Earithmetic_2E_2B V8b) \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \\
& (ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL (ap \\
& c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) V9e)) (ap (ap \\
& (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) (ap (ap V6f V8b) (ap c_2Earithmetic_2EODD \\
& V7x))) (ap (ap c_2Earithmetic_2E_2B V9e) V10y))))))))))
\end{aligned} \tag{37}$$

Theorem 1

$$\begin{aligned}
& ((\forall V0f \in ((2^2)^{ty_2Enum_2Enum}).(\forall V1x \in ty_2Enum_2Enum. \\
& (\forall V2b \in ty_2Enum_2Enum.(\forall V3e \in ty_2Enum_2Enum.(\\
\forall V4y \in ty_2Enum_2Enum.((ap (ap (ap (ap (ap (ap c_2Enumeral_bit_2EBIT_MODF \\
& c_2Enum_2E0) V0f) V1x) V2b) V3e) V4y) = V4y)))))) \wedge ((\forall V5n \in \\
& ty_2Enum_2Enum.(\forall V6f \in ((2^2)^{ty_2Enum_2Enum}).(\forall V7b \in \\
& ty_2Enum_2Enum.(\forall V8e \in ty_2Enum_2Enum.(\forall V9y \in ty_2Enum_2Enum. \\
((ap (ap (ap (ap (ap (ap c_2Enumeral_bit_2EBIT_MODF (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 V5n))) V6f) c_2Enum_2E0) V7b) (ap c_2Earithmetic_2ENUMERAL \\
& V8e)) V9y) = (ap (ap (ap (ap (ap (ap c_2Enumeral_bit_2EBIT_MODF \\
& (ap (ap c_2Earithmetic_2E_2D (ap c_2Earithmetic_2ENUMERAL (ap \\
& c_2Earithmetic_2EBIT1 V5n))) (ap c_2Earithmetic_2ENUMERAL (\\
& ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) V6f) c_2Enum_2E0) \\
& (ap (ap c_2Earithmetic_2E_2B V7b) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Enumeral_2EiDUB V8e))) (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) \\
& (ap (ap V6f V7b) c_2Ebool_2EF)) (ap (ap c_2Earithmetic_2E_2B (ap \\
& c_2Earithmetic_2ENUMERAL V8e)) V9y)) V9y)))))) \wedge ((\forall V10n \in \\
& ty_2Enum_2Enum.(\forall V11f \in ((2^2)^{ty_2Enum_2Enum}).(\forall V12b \in \\
& ty_2Enum_2Enum.(\forall V13e \in ty_2Enum_2Enum.(\forall V14y \in \\
& ty_2Enum_2Enum.((ap (ap (ap (ap (ap (ap c_2Enumeral_bit_2EBIT_MODF \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 V10n))) \\
& V11f) c_2Enum_2E0) V12b) (ap c_2Earithmetic_2ENUMERAL V13e)) \\
& V14y) = (ap (ap (ap (ap (ap (ap c_2Enumeral_bit_2EBIT_MODF (ap \\
& c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V10n))) \\
V11f) c_2Enum_2E0) (ap (ap c_2Earithmetic_2E_2B V12b) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Enumeral_2EiDUB V13e))) (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) \\
& (ap (ap V11f V12b) c_2Ebool_2EF)) (ap (ap c_2Earithmetic_2E_2B \\
& (ap c_2Earithmetic_2ENUMERAL V13e)) V14y)) V14y)))))) \wedge ((\forall V15n \in \\
& ty_2Enum_2Enum.(\forall V16f \in ((2^2)^{ty_2Enum_2Enum}).(\forall V17x \in \\
& ty_2Enum_2Enum.(\forall V18b \in ty_2Enum_2Enum.(\forall V19e \in \\
& ty_2Enum_2Enum.(\forall V20y \in ty_2Enum_2Enum.((ap (ap (ap (ap \\
& (ap (ap c_2Enumeral_bit_2EBIT_MODF (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 V15n))) V16f) (ap c_2Earithmetic_2ENUMERAL \\
& V17x)) V18b) (ap c_2Earithmetic_2ENUMERAL V19e)) V20y) = (ap (ap \\
& (ap (ap (ap (ap c_2Enumeral_bit_2EBIT_MODF (ap (ap c_2Earithmetic_2E_2D \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V15n))) \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \\
& V16f) (ap c_2Earithmetic_2EDIV2 (ap c_2Earithmetic_2ENUMERAL \\
& V17x))) (ap (ap c_2Earithmetic_2E_2B V18b) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Enumeral_2EiDUB V19e))) (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) \\
& (ap (ap V16f V18b) (ap c_2Earithmetic_2EODD V17x))) (ap (ap c_2Earithmetic_2E_2B \\
& (ap c_2Earithmetic_2ENUMERAL V19e)) V20y)) V20y)))))) \wedge ((\forall V21n \in \\
& ty_2Enum_2Enum.(\forall V22f \in ((2^2)^{ty_2Enum_2Enum}).(\forall V23x \in \\
& ty_2Enum_2Enum.(\forall V24b \in ty_2Enum_2Enum.(\forall V25e \in \\
& ty_2Enum_2Enum.(\forall V26y \in ty_2Enum_2Enum.((ap (ap (ap (ap \\
& (ap (ap c_2Enumeral_bit_2EBIT_MODF (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT2 V21n))) V22f) (ap c_2Earithmetic_2ENUMERAL \\
& V23x)) V24b) (ap c_2Earithmetic_2ENUMERAL V25e)) V26y) = (ap (ap \\
& (ap (ap (ap (ap c_2Enumeral_bit_2EBIT_MODF (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 V21n))) V22f) (ap c_2Earithmetic_2EDIV2 \\
& (ap c_2Earithmetic_2ENUMERAL V23x))) (ap (ap c_2Earithmetic_2E_2B \\
& V24b) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO)))) (ap c_2Earithmetic_2ENUMERAL (ap \\
& c_2Enumeral_2EiDUB V25e))) (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) \\
& (ap (ap V22f V24b) (ap c_2Earithmetic_2EODD V23x))) (ap (ap c_2Earithmetic_2E_2B \\
& (ap c_2Earithmetic_2ENUMERAL V25e)) V26y)) V26y)))))))))
\end{aligned}$$