thm_2Enumeral__bit_2ENUMERAL_DIV_2EXP (TMN5WizWFdYDbJhFhStY1vuTRg1hf3E9Wkp)

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Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p \ (ap \ P \ x))$ **then** $(the \ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o \ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap \ V0P \ (ap \ (\text{c\_2Emin\_2E\_40} \ A$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty \ ty\_2Enum\_2Enum \qquad (1)$$

Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(2)$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \qquad (3)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \qquad (4)$$

Definition 4 *We define* c_2Enum_2E0 *to be* $(ap \ c\_2Enum\_2EABS\_\_num \ c\_2Enum\_2EZERO\_\_REP)$.

Definition 5 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \qquad (5)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \qquad (6)$$

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Definition 6 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D $(2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 7 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ ($c_2Emin_2E_3D $(2^{A\_27a}$

Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
(7)

Definition 9 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 10 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
(8)

Definition 11 *We define* c_2Ebit_2EDIV__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 12 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 13 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum})$$
(9)

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum})$$
(10)

Definition 14 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t)).$

Definition 15 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ $c_2Emin_2E_3D_3D_3E $V0t)\ $c_2Ebool_2E

Definition 16 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Definition 17 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 18 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 19 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 20 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 21 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 22 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ ($c_2Ebool_2E

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$

(11)

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$

(12)

Definition 23 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Definition 24 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 25 *We define* c_2Earithmetic_2EDIV2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 26 *We define* c_2Enumeral_bit_2EiDIV2 *to be* c_2Earithmetic_2EDIV2.

Let $c\_2Enumeral\_bit\_2ESFUNPOW : \iota$ be given. Assume the following.

$$c\_2Enumeral\_bit\_2ESFUNPOW \in (((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{(ty\_2Enum\_2E}$$

(13)

Assume the following.

$$((\forall V0m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP$$
$$V0m)\ \mathsf{c\_2Enum\_2E0}) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$\mathsf{c\_2Earithmetic\_2EZERO})))) \wedge (\forall V1m \in ty\_2Enum\_2Enum.(\forall V2n \in$$
$$ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ V1m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}$$
$$V2n)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V1m)\ (ap\ (ap\ c\_2Earithmetic\_2EEXP$$
$$V1m)\ V2n))))))$$

(14)

Assume the following.

$$((ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})) =$$
$$(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$\mathsf{c\_2Earithmetic\_2EZERO}))))$$

(15)

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((V0m = \mathsf{c\_2Enum\_2E0}) \vee (\exists V1n \in$$
$$ty\_2Enum\_2Enum.(V0m = (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)))))$$

(16)

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$p\ (ap\ (ap\ \mathsf{c\_2Eprim\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ (ap\ (ap\ c\_2Earithmetic\_2EEXP$$
$$(ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))\ V0m)))))$$

(17)

3

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1k \in ty\_2Enum\_2Enum.($$
$$\forall V2q \in ty\_2Enum\_2Enum.((\exists V3r \in ty\_2Enum\_2Enum.($$
$$(V1k = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A$$
$$V2q)\ V0n))\ V3r))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V3r)\ V0n))))\Rightarrow($$
$$(ap\ (ap\ c\_2Earithmetic\_2EDIV\ V1k)\ V0n) = V2q)))))$$

$$(18)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$\mathsf{c\_2Enum\_2E0})\ V0n))\Rightarrow((ap\ (ap\ c\_2Earithmetic\_2EDIV\ \mathsf{c\_2Enum\_2E0})$$
$$V0n) = \mathsf{c\_2Enum\_2E0})))$$

$$(19)$$

Assume the following.

$$(\forall V0q \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EDIV\ V0q)$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))) =$$
$$V0q))$$

$$(20)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V0m))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$\mathsf{c\_2Enum\_2E0})\ V1n)))\Rightarrow(\forall V2x \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EDIV$$
$$(ap\ (ap\ c\_2Earithmetic\_2EDIV\ V2x)\ V0m))\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2EDIV$$
$$V2x)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n)))))))$$

$$(21)$$

Assume the following.
$$True \qquad (22)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p$$
$$V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2))))))$$

$$(23)$$

Assume the following.

$$(\forall V0t \in 2.(False\Rightarrow(p\ V0t))) \qquad (24)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t)\vee(\neg(p\ V0t)))) \qquad (25)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0t \in 2.((\forall V1x \in$$
$$A\_27a.(p\ V0t))\Leftrightarrow(p\ V0t)))$$

$$(26)$$

4

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow$$
$$(p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \quad (27)$$
$$(((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))))$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \quad (28)$$
$$True))$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \quad (29)$$
$$A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x))))$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow$$
$$(p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (\quad (30)$$
$$p\ V0t)))))))$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in$$
$$A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)$$
$$V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})$$
$$V0t1)\ V1t2) = V1t2))))$$

$$(31)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow$$
$$((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (32)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2.$$
$$(\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a.$$
$$(\forall V5y\_27 \in A\_27a.((((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge$$
$$((\neg (p\ V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)$$
$$V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V1Q)\ V3x\_27)$$
$$V5y\_27)))))))))$$

$$(33)$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p\ (ap\ V0P\ \mathsf{c\_2Enum\_2E0})) \wedge$$
$$(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ \mathsf{c\_2Enum\_2ESUC}$$
$$V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ V2n)))))$$

$$(34)$$

Assume the following.

$((\forall V0n \in ty\_2Enum\_2Enum.((ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_2B}$
$\mathsf{c\_2Enum\_2E0}) \ V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap$
$(ap \ c\_2Earithmetic\_2E\_2B \ V1n) \ \mathsf{c\_2Enum\_2E0}) = V1n)) \wedge ((\forall V2n \in$
$ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap \ (ap \ c\_2Earithmetic\_2E\_2B$
$(ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL} \ V2n)) \ (ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL}$
$V3m)) = (ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL} \ (ap \ \mathsf{c\_2Enumeral\_2EiZ} \ (ap$
$(ap \ c\_2Earithmetic\_2E\_2B \ V2n) \ V3m)))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum.$
$((ap \ (ap \ c\_2Earithmetic\_2E\_2A \ \mathsf{c\_2Enum\_2E0}) \ V4n) = \mathsf{c\_2Enum\_2E0})) \wedge$
$((\forall V5n \in ty\_2Enum\_2Enum.((ap \ (ap \ c\_2Earithmetic\_2E\_2A$
$V5n) \ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0})) \wedge ((\forall V6n \in ty\_2Enum\_2Enum.$
$(\forall V7m \in ty\_2Enum\_2Enum.((ap \ (ap \ c\_2Earithmetic\_2E\_2A \ ($
$ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL} \ V6n)) \ (ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL}$
$V7m)) = (ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2A$
$V6n) \ V7m))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap \ (ap \ c\_2Earithmetic\_2E\_2D$
$\mathsf{c\_2Enum\_2E0}) \ V8n) = \mathsf{c\_2Enum\_2E0})) \wedge ((\forall V9n \in ty\_2Enum\_2Enum.$
$((ap \ (ap \ c\_2Earithmetic\_2E\_2D \ V9n) \ \mathsf{c\_2Enum\_2E0}) = V9n)) \wedge ((\forall V10n \in$
$ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap \ (ap \ c\_2Earithmetic\_2E\_2D$
$(ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL} \ V10n)) \ (ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL}$
$V11m)) = (ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2D$
$V10n) \ V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap \ (ap \ c\_2Earithmetic\_2EEXP$
$\mathsf{c\_2Enum\_2E0}) \ (ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL} \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT1}$
$V12n))) = \mathsf{c\_2Enum\_2E0})) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap$
$(ap \ c\_2Earithmetic\_2EEXP \ \mathsf{c\_2Enum\_2E0}) \ (ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL}$
$(ap \ \mathsf{c\_2Earithmetic\_2EBIT2} \ V13n))) = \mathsf{c\_2Enum\_2E0})) \wedge ((\forall V14n \in$
$ty\_2Enum\_2Enum.((ap \ (ap \ c\_2Earithmetic\_2EEXP \ V14n) \ \mathsf{c\_2Enum\_2E0}) =$
$(ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL} \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT1} \ \mathsf{c\_2Earithmetic\_2EZERO})))) \wedge$
$((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum.$
$((ap \ (ap \ c\_2Earithmetic\_2EEXP \ (ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL} \ V15n))$
$(ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL} \ V16m)) = (ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL}$
$(ap \ (ap \ c\_2Earithmetic\_2EEXP \ V15n) \ V16m))))) \wedge (((ap \ \mathsf{c\_2Enum\_2ESUC}$
$\mathsf{c\_2Enum\_2E0}) = (ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL} \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT1}$
$\mathsf{c\_2Earithmetic\_2EZERO}))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum.($
$(ap \ \mathsf{c\_2Enum\_2ESUC} \ (ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL} \ V17n)) = (ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL}$
$(ap \ \mathsf{c\_2Enum\_2ESUC} \ V17n)))) \wedge (((ap \ \mathsf{c\_2Eprim\_\_rec\_2EPRE} \ \mathsf{c\_2Enum\_2E0}) =$
$\mathsf{c\_2Enum\_2E0}) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap \ \mathsf{c\_2Eprim\_\_rec\_2EPRE}$
$(ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL} \ V18n)) = (ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL}$
$(ap \ \mathsf{c\_2Eprim\_\_rec\_2EPRE} \ V18n)))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum.$
$(((ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL} \ V19n) = \mathsf{c\_2Enum\_2E0}) \Leftrightarrow (V19n = \mathsf{c\_2Earithmetic\_2EZERO}))) \wedge$
$((\forall V20n \in ty\_2Enum\_2Enum.((\mathsf{c\_2Enum\_2E0} = (ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL}$
$V20n)) \Leftrightarrow (V20n = \mathsf{c\_2Earithmetic\_2EZERO}))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum.$
$(\forall V22m \in ty\_2Enum\_2Enum.(((ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL}$
$V21n) = (ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL} \ V22m)) \Leftrightarrow (V21n = V22m)))) \wedge$
$((\forall V23n \in ty\_2Enum\_2Enum.((p \ (ap \ (ap \ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$
$V23n) \ \mathsf{c\_2Enum\_2E0})) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum.$
$((p \ (ap \ (ap \ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \ \mathsf{c\_2Enum\_2E0}) \ (ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL}$
$V24n))) \Leftrightarrow (p \ (ap \ (ap \ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \ \mathsf{c\_2Earithmetic\_2EZERO})$
$V24n)))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum.$
$((p \ (ap \ (ap \ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \ (ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL}$
$V25n)) \ (ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL} \ V26m))) \Leftrightarrow (p \ (ap \ (ap \ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$
$V25n) \ V26m))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3E}$
$\mathsf{c\_2Enum\_2E0}) \ V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum.$
$((p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3E} \ (ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL}$
$V28n)) \ \mathsf{c\_2Enum\_2E0})) \Leftrightarrow (p \ (ap \ (ap \ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \ \mathsf{c\_2Earithmetic\_2EZERO})$
$V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum.$
$((p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3E} \ (ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL}$
$V29n)) \ (ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL} \ V30m))) \Leftrightarrow (p \ (ap \ (ap \ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$
$V30m) \ V29n))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$
$\mathsf{c\_2Enum\_2E0}) \ V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum.$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$
$$((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n)))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n)))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V0n)\ \mathsf{c\_2Earithmetic\_2EZERO}))\Leftrightarrow False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))\Leftrightarrow$$
$$(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))\Leftrightarrow$$
$$(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))\Leftrightarrow$$
$$(\neg(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1m)\ V0n))))\wedge((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))\Leftrightarrow$$
$$(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m)))))))))))$$

$$(36)$$

Assume the following.

$$((\forall V0f \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}).(\forall V1x \in$$
$$ty\_2Enum\_2Enum.((ap\ (ap\ (ap\ c\_2Enumeral\_\_bit\_2ESFUNPOW\ V0f)$$
$$\mathsf{c\_2Enum\_2E0})\ V1x) = V1x)))\wedge(\forall V2f \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}).$$
$$(\forall V3n \in ty\_2Enum\_2Enum.(\forall V4x \in ty\_2Enum\_2Enum.($$
$$(ap\ (ap\ (ap\ c\_2Enumeral\_\_bit\_2ESFUNPOW\ V2f)\ (ap\ \mathsf{c\_2Enum\_2ESUC}$$
$$V3n))\ V4x) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Enum\_2Enum)\ (ap\ (ap$$
$$(\mathsf{c\_2Emin\_2E\_3D}\ ty\_2Enum\_2Enum)\ V4x)\ \mathsf{c\_2Enum\_2E0}))\ \mathsf{c\_2Enum\_2E0})$$
$$(ap\ (ap\ (ap\ c\_2Enumeral\_\_bit\_2ESFUNPOW\ V2f)\ V3n)\ (ap\ V2f\ V4x))))))))))$$

$$(37)$$

Theorem 1

$$((\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ \mathsf{c\_2Ebit\_2EDIV\_\_2EXP}\ V0n)$$
$$\mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0}))\wedge(\forall V1n \in ty\_2Enum\_2Enum.($$
$$\forall V2x \in ty\_2Enum\_2Enum.((ap\ (ap\ \mathsf{c\_2Ebit\_2EDIV\_\_2EXP}\ V1n)$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V2x)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ (ap\ (ap\ c\_2Enumeral\_\_bit\_2ESFUNPOW\ \mathsf{c\_2Enumeral\_\_bit\_2EiDIV2})$$
$$V1n)\ V2x)))))))$$