

thm_2Enumeral_bit_2EiBITWISE (TMccRu1hDxzDkjybk46asCqK87ruKyr6T)

October 26, 2020

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define c_2Ebool_2E21 to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (ap\ (ap\ (c_2Emin_2E3D\ (2^{2^m}))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Let $c_2Earithmetic_2E2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{6}$$

Definition 7 We define `c_2Earithmetic_2EBIT2` to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EBIT2))$

Definition 8 We define `c_2Earithmetic_2ENUMERAL` to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let `c_2Earithmetic_2EDIV` : ι be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (7)$$

Definition 9 We define `c_2Earithmetic_2EDIV2` to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EDIV2))$

Definition 10 We define `c_2Earithmetic_2EBIT1` to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EBIT1))$

Let `c_2Earithmetic_2EEXP` : ι be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (8)$$

Definition 11 We define `c_2Ebit_2EDIV_2EXP` to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let `c_2Earithmetic_2E_2D` : ι be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (9)$$

Let `c_2Earithmetic_2EMOD` : ι be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (10)$$

Definition 12 We define `c_2Ebit_2EMOD_2EXP` to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 13 We define `c_2Ebit_2EBITS` to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V2$

Definition 14 We define `c_2Ebit_2EBIT` to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Definition 15 We define `c_2Ebool_2EF` to be $(ap (c_2Ebool_2E_21) 2) (\lambda V0t \in 2.V0t)$.

Definition 16 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow Q)$ of type ι .

Definition 17 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2)))$

Definition 18 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A))$ of type $\iota \Rightarrow \iota$.

Definition 19 We define `c_2Ebool_2ECOND` to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap$

Definition 20 We define `c_2Ebit_2ESBIT` to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap (ap (ap (c_2Ebool_2E_21) 2) 2) 2)$

Definition 21 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2)))$

Definition 22 We define c_Ebool_ELET to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0f \in (A_27b^{A-27a}). (\lambda V1x \in A_27$

Definition 23 We define $c_Ebool_E_7E$ to be $(\lambda V0t \in 2. (ap (ap c_Emin_E_3D_3D_3E V0t) c_Ebool_E_7E$

Let $c_Earithmetic_EEVEN : \iota$ be given. Assume the following.

$$c_Earithmetic_EEVEN \in (2^{ty_2Enum_2Enum}) \quad (11)$$

Let $c_Earithmetic_EODD : \iota$ be given. Assume the following.

$$c_Earithmetic_EODD \in (2^{ty_2Enum_2Enum}) \quad (12)$$

Definition 24 We define $c_Ebool_E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap V0P (ap (c_Emin_E_40$

Definition 25 We define $c_Eprim_rec_E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 26 We define $c_Earithmetic_E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 27 We define $c_Earithmetic_E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 28 We define $c_Earithmetic_E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 29 We define $c_Eprim_rec_EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum. (ap (ap (ap (c_Ebool_E_7E$

Let $c_Earithmetic_E_2A : \iota$ be given. Assume the following.

$$c_Earithmetic_E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (13)$$

Definition 30 We define $c_Enumeral_EiSUC$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap c_Eenum_ESUC (ap$

Definition 31 We define $c_Enumeral_EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Definition 32 We define $c_Enumeral_EiDUB$ to be $\lambda V0x \in ty_2Enum_2Enum. (ap (ap c_Earithmetic_E_7E$

Let $c_Ebit_EBITWISE : \iota$ be given. Assume the following.

$$c_Ebit_EBITWISE \in (((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{((2^2)^2)})^{ty_2Enum_2Enum}) \quad (14)$$

Definition 33 We define $c_Enumeral_bit_EiBITWISE$ to be $c_Ebit_EBITWISE$.

Assume the following.

$$\begin{aligned} & ((\forall V0m \in ty_2Enum_2Enum. ((ap (ap c_Earithmetic_EEXP \\ & V0m) c_Eenum_E0) = (ap c_Earithmetic_ENUMERAL (ap c_Earithmetic_EBIT1 \\ & c_Earithmetic_EZERO)))) \wedge (\forall V1m \in ty_2Enum_2Enum. (\forall V2n \in \\ & ty_2Enum_2Enum. ((ap (ap c_Earithmetic_EEXP V1m) (ap c_Eenum_ESUC \\ & V2n)) = (ap (ap c_Earithmetic_E_2A V1m) (ap (ap c_Earithmetic_EEXP \\ & V1m) V2n)))))) \end{aligned} \quad (15)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m)) \quad (16)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B V1n) V0m)))) \quad (17)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B V1n) V0m)))) \quad (18)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B V0m) V1n) V2p) = (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p)))))) \quad (19)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)) V0m) = V0m)) \quad (20)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2E_2B V0m) V1n) V2p) = (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V2p)) (ap (ap c_2Earithmetic_2E_2A V1n) V2p)))))) \quad (21)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in ((A.27a^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}). \\
& \quad (\forall V1g \in (A.27a^{ty_2Enum_2Enum}).((\forall V2n \in ty_2Enum_2Enum. \\
& \quad ((ap\ V1g\ (ap\ c.2Enum_2ESUC\ V2n)) = (ap\ (ap\ V0f\ V2n)\ (ap\ c.2Enum_2ESUC \\
& \quad V2n)))) \Leftrightarrow ((\forall V3n \in ty_2Enum_2Enum.((ap\ V1g\ (ap\ c.2Earithmetic_2ENUMERAL \\
& \quad (ap\ c.2Earithmetic_2EBIT1\ V3n))) = (ap\ (ap\ V0f\ (ap\ (ap\ c.2Earithmetic_2E_2D \\
& \quad (ap\ c.2Earithmetic_2ENUMERAL\ (ap\ c.2Earithmetic_2EBIT1\ V3n))) \\
& \quad (ap\ c.2Earithmetic_2ENUMERAL\ (ap\ c.2Earithmetic_2EBIT1\ c.2Earithmetic_2EZERO)))))) \\
& \quad (ap\ c.2Earithmetic_2ENUMERAL\ (ap\ c.2Earithmetic_2EBIT1\ V3n)))))) \wedge \\
& \quad (\forall V4n \in ty_2Enum_2Enum.((ap\ V1g\ (ap\ c.2Earithmetic_2ENUMERAL \\
& \quad (ap\ c.2Earithmetic_2EBIT2\ V4n))) = (ap\ (ap\ V0f\ (ap\ c.2Earithmetic_2ENUMERAL \\
& \quad (ap\ c.2Earithmetic_2EBIT1\ V4n)))\ (ap\ c.2Earithmetic_2ENUMERAL \\
& \quad (ap\ c.2Earithmetic_2EBIT2\ V4n)))))))))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0op \in ((2^2)^2).(\forall V1x \in ty_2Enum_2Enum.(\forall V2y \in \\
& \quad ty_2Enum_2Enum.((ap\ (ap\ (ap\ (ap\ c.2Ebit_2EBITWISE\ c.2Enum_2E0) \\
& \quad V0op)\ V1x)\ V2y) = c.2Enum_2E0)))) \wedge (\forall V3n \in ty_2Enum_2Enum. \\
& \quad (\forall V4op \in ((2^2)^2).(\forall V5x \in ty_2Enum_2Enum.(\forall V6y \in \\
& \quad ty_2Enum_2Enum.((ap\ (ap\ (ap\ (ap\ c.2Ebit_2EBITWISE\ (ap\ c.2Enum_2ESUC \\
& \quad V3n))\ V4op)\ V5x)\ V6y) = (ap\ (ap\ c.2Earithmetic_2E_2B\ (ap\ (ap\ (ap\ (\\
& \quad ap\ c.2Ebit_2EBITWISE\ V3n)\ V4op)\ V5x)\ V6y))\ (ap\ (ap\ c.2Ebit_2ESBIT \\
& \quad (ap\ (ap\ V4op\ (ap\ (ap\ c.2Ebit_2EBIT\ V3n)\ V5x))\ (ap\ (ap\ c.2Ebit_2EBIT \\
& \quad V3n)\ V6y)))\ V3n)))))))))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum.(\forall V1op \in ((2^2)^2).(\forall V2a \in \\
& \quad ty_2Enum_2Enum.(\forall V3b \in ty_2Enum_2Enum.((ap\ (ap\ (ap\ (ap \\
& \quad c.2Ebit_2EBITWISE\ (ap\ c.2Enum_2ESUC\ V0n))\ V1op)\ V2a)\ V3b) = (ap \\
& \quad (ap\ c.2Earithmetic_2E_2B\ (ap\ (ap\ c.2Earithmetic_2E_2A\ (ap\ c.2Earithmetic_2ENUMERAL \\
& \quad (ap\ c.2Earithmetic_2EBIT2\ c.2Earithmetic_2EZERO))))\ (ap\ (ap\ (\\
& \quad ap\ (ap\ c.2Ebit_2EBITWISE\ V0n)\ V1op)\ (ap\ (ap\ c.2Earithmetic_2EDIV \\
& \quad V2a)\ (ap\ c.2Earithmetic_2ENUMERAL\ (ap\ c.2Earithmetic_2EBIT2 \\
& \quad c.2Earithmetic_2EZERO))))\ (ap\ (ap\ c.2Earithmetic_2EDIV\ V3b) \\
& \quad (ap\ c.2Earithmetic_2ENUMERAL\ (ap\ c.2Earithmetic_2EBIT2\ c.2Earithmetic_2EZERO)))))) \\
& \quad (ap\ (ap\ c.2Ebit_2ESBIT\ (ap\ (ap\ V1op\ (ap\ c.2Earithmetic_2EODD\ V2a)) \\
& \quad (ap\ c.2Earithmetic_2EODD\ V3b)))\ c.2Enum_2E0))))))
\end{aligned} \tag{24}$$

Assume the following.

$$True \tag{25}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{26}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (28)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0f \in (A_27b^{A_27a}). (\forall V1x \in A_27a. ((ap\ (ap\ (c_2Ebool_2ELET \\ & A_27a\ A_27b)\ V0f)\ V1x) = (ap\ V0f\ V1x)))) \end{aligned} \quad (29)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (30)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (31)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p\ V0t)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in \\ & A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1)\ \\ & V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ \\ & V0t1)\ V1t2) = V1t2)))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\ & (\forall V5y_27 \in A_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ \\ & V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27)\ \\ & V5y_27)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow ((\forall V0t1 \in A_{27a}. (\forall V1t2 \in \\
& A_{27a}. ((\text{ap } (\text{ap } (\text{ap } (\text{c_2Ebool_2ECOND } A_{27a}) \text{ c_2Ebool_2ET}) V0t1) \\
& V1t2) = V0t1))) \wedge (\forall V2t1 \in A_{27a}. (\forall V3t2 \in A_{27a}. ((\text{ap} \\
& (\text{ap } (\text{ap } (\text{c_2Ebool_2ECOND } A_{27a}) \text{ c_2Ebool_2EF}) V2t1) V3t2) = V3t2))))))
\end{aligned}
\tag{36}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (\\
& ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (\\
& ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (\\
& ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap \\
& c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC \\
& (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ c_2Earithmetic_2EZERO)) = (\\
& ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (\\
& ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT1 \\
& V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m))))))))))))))))))))))))))))))
\end{aligned}$$

(38)

Theorem 1

$$\begin{aligned}
& ((\forall V0opr \in ((2^2)^2).(\forall V1a \in ty_2Enum_2Enum.(\forall V2b \in \\
& ty_2Enum_2Enum.((ap (ap (ap (ap c_2Enumeral_bit_2EiBITWISE \\
& c_2Enum_2E0) V0opr) V1a) V2b) = c_2Earithmetic_2EZERO)))) \wedge ((\\
& \forall V3x \in ty_2Enum_2Enum.(\forall V4opr \in ((2^2)^2).(\forall V5a \in \\
& ty_2Enum_2Enum.(\forall V6b \in ty_2Enum_2Enum.((ap (ap (ap (ap \\
& c_2Enumeral_bit_2EiBITWISE (ap c_2Earithmetic_2ENUMERAL (\\
& ap c_2Earithmetic_2EBIT1 V3x))) V4opr) V5a) V6b) = (ap (ap (c_2Ebool_2ELET \\
& ty_2Enum_2Enum ty_2Enum_2Enum) (\lambda V7w \in ty_2Enum_2Enum.(ap \\
& (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) (ap (ap V4opr (ap c_2Earithmetic_2EODD \\
& V5a)) (ap c_2Earithmetic_2EODD V6b))) (ap c_2Earithmetic_2EBIT1 \\
& V7w)) (ap c_2Enumeral_2EiDUB V7w)))) (ap (ap (ap (ap c_2Enumeral_bit_2EiBITWISE \\
& (ap (ap c_2Earithmetic_2E_2D (ap c_2Earithmetic_2ENUMERAL (ap \\
& c_2Earithmetic_2EBIT1 V3x))) (ap c_2Earithmetic_2ENUMERAL (\\
& ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) V4opr) (\\
& ap c_2Earithmetic_2EDIV2 V5a)) (ap c_2Earithmetic_2EDIV2 V6b)))))) \wedge \\
& (\forall V8x \in ty_2Enum_2Enum.(\forall V9opr \in ((2^2)^2).(\forall V10a \in \\
& ty_2Enum_2Enum.(\forall V11b \in ty_2Enum_2Enum.((ap (ap (ap (ap \\
& c_2Enumeral_bit_2EiBITWISE (ap c_2Earithmetic_2ENUMERAL (\\
& ap c_2Earithmetic_2EBIT2 V8x))) V9opr) V10a) V11b) = (ap (ap (c_2Ebool_2ELET \\
& ty_2Enum_2Enum ty_2Enum_2Enum) (\lambda V12w \in ty_2Enum_2Enum.(\\
& ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) (ap (ap V9opr (ap c_2Earithmetic_2EODD \\
& V10a)) (ap c_2Earithmetic_2EODD V11b))) (ap c_2Earithmetic_2EBIT1 \\
& V12w)) (ap c_2Enumeral_2EiDUB V12w)))) (ap (ap (ap (ap c_2Enumeral_bit_2EiBITWISE \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V8x))) \\
& V9opr) (ap c_2Earithmetic_2EDIV2 V10a)) (ap c_2Earithmetic_2EDIV2 \\
& V11b)))))))))
\end{aligned}$$