

thm_2Enumeral_bit_2Enumeral_imod_2exp
(TMMA-
JYNQv28GJzZRReeNGMb4vY5g9E59GN5W)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \ x) \text{ of type } \iota \Rightarrow \iota.$

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota.$

Definition 3 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P \text{ (ap } (c_2Emin_2E_40 \ A \ P))$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty } ty_2Enum_2Enum \tag{1}$$

Let `c_2Earithmetic_2E_2D` : ι be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{2}$$

Let `c_2Earithmetic_2EDIV` : ι be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{3}$$

Let `c_2Earithmetic_2E_2A` : ι be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{4}$$

Definition 4 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (c_2Emin_2E_3D \ (2^2)) \ (\lambda V0x \in 2.V0x)) \ (\lambda V1x \in 2.V1x))$

Definition 5 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (c_2Emin_2E_3D \ (2^{A-27a})) \ (\lambda V1x \in 2.V1x))$

Definition 6 We define `c_2Earithmetic_2ENUMERAL` to be $\lambda V0x \in ty_2Enum_2Enum. V0x.$

Let `c_2Earithmetic_2EEXP` : ι be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{5}$$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 7 We define $c_2Ebool_2EBOUNDED$ to be $(\lambda V0v \in 2.c_2Ebool_2ET)$.

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (7)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (8)$$

Definition 8 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (9)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (10)$$

Definition 9 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ m)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (11)$$

Definition 10 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ n))$

Definition 11 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ n))$

Definition 12 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Definition 13 We define $c_2Enumeral_2EiDUB$ to be $\lambda V0x \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ x))$

Definition 14 We define $c_2Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 15 We define $c_2Enumeral_bit_2EiMOD_2EXP$ to be $c_2Ebit_2EMOD_2EXP$.

Definition 16 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 17 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 18 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_21\ 2))$

Definition 19 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V0t))))$

Definition 20 We define $c_Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Definition 21 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_Ebool_2E_21 2) (\lambda V2t \in$

Assume the following.

$$\begin{aligned} & ((\forall V0n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmic_2E_2B \\ & c_2Enum_2E0) V0n) = V0n)) \wedge (\forall V1m \in ty_2Enum_2Enum. (\forall V2n \in \\ & ty_2Enum_2Enum. ((ap (ap c_2Earithmic_2E_2B (ap c_2Enum_2ESUC \\ & V1m)) V2n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmic_2E_2B V1m) \\ & V2n))))))) \end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned} & ((\forall V0m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmic_2EEXP \\ & V0m) c_2Enum_2E0) = (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 \\ & c_2Earithmic_2EZERO)))) \wedge (\forall V1m \in ty_2Enum_2Enum. (\forall V2n \in \\ & ty_2Enum_2Enum. ((ap (ap c_2Earithmic_2EEXP V1m) (ap c_2Enum_2ESUC \\ & V2n)) = (ap (ap c_2Earithmic_2E_2A V1m) (ap (ap c_2Earithmic_2EEXP \\ & V1m) V2n)))))) \end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned} & ((ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)) = \\ & (ap c_2Enum_2ESUC c_2Enum_2E0)) \end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned} & ((ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT2 c_2Earithmic_2EZERO)) = \\ & (ap c_2Enum_2ESUC (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 \\ & c_2Earithmic_2EZERO)))) \end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmic_2E_2B V0m) \\ & (ap (ap c_2Earithmic_2E_2B V1n) V2p)) = (ap (ap c_2Earithmic_2E_2B \\ & (ap (ap c_2Earithmic_2E_2B V0m) V1n)) V2p)))))) \end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. ((V0m = c_2Enum_2E0) \vee (\exists V1n \in \\ & ty_2Enum_2Enum. (V0m = (ap c_2Enum_2ESUC V1n)))))) \end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0m) = (ap (ap \\ & c_2Earithmic_2E_2B V0m) (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 \\ & c_2Earithmic_2EZERO)))))) \end{aligned} \tag{18}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D (ap c_2Enum_2ESUC V0m)) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) = V0m)) \quad (19)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0)) \quad (20)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) = V0m)) \quad (21)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A V0m) V1n) = (ap (ap c_2Earithmetic_2E_2A V1n) V0m)))) \quad (22)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) V0n) = (ap (ap c_2Earithmetic_2E_2B V0n) V0n))) \quad (23)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap (ap c_2Earithmetic_2EEXP (ap c_2Enum_2ESUC V1n)) V0m)))))) \quad (24)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) (ap (ap c_2Earithmetic_2E_2A V0m) V2p))) \Leftrightarrow ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V0m)) \wedge (p (ap (ap c_2Eprim_rec_2E_3C V1n) V2p))))))) \quad (25)$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad c_2Enum_2E0) V0n)) \Rightarrow (\forall V1k \in ty_2Enum_2Enum. ((V1k = (ap (\\
& \quad ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A (ap (ap c_2Earithmetic_2EDIV \\
& \quad V1k) V0n)) V0n)) (ap (ap c_2Earithmetic_2EMOD V1k) V0n))) \wedge (p (ap \\
& \quad (ap c_2Eprim_rec_2E_3C (ap (ap c_2Earithmetic_2EMOD V1k) V0n)) \\
& \quad V0n))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& (\forall V0k \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EMOD V0k) \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = \\
& \quad c_2Enum_2E0))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1k \in ty_2Enum_2Enum. (\\
& (p (ap (ap c_2Eprim_rec_2E_3C V1k) V0n)) \Rightarrow ((ap (ap c_2Earithmetic_2EMOD \\
& \quad V1k) V0n) = V1k))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad c_2Enum_2E0) V0n)) \Rightarrow ((ap (ap c_2Earithmetic_2EMOD c_2Enum_2E0) \\
& \quad V0n) = c_2Enum_2E0)))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1r \in ty_2Enum_2Enum. (\\
& (p (ap (ap c_2Eprim_rec_2E_3C V1r) V0n)) \Rightarrow (\forall V2q \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Earithmetic_2EMOD (ap (ap c_2Earithmetic_2E_2B (ap \\
& \quad (ap c_2Earithmetic_2E_2A V2q) V0n)) V1r)) V0n) = V1r))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad c_2Enum_2E0) V0n)) \Rightarrow (\forall V1j \in ty_2Enum_2Enum. (\forall V2k \in \\
& ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EMOD (ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap (ap c_2Earithmetic_2EMOD V1j) V0n)) (ap (ap c_2Earithmetic_2EMOD \\
& \quad V2k) V0n))) V0n) = (ap (ap c_2Earithmetic_2EMOD (ap (ap c_2Earithmetic_2E_2B \\
& \quad V1j) V2k)) V0n))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1p \in ty_2Enum_2Enum. (\\
& \quad \forall V2q \in ty_2Enum_2Enum. (((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad \quad c_2Enum_2E0) V0n)) \wedge (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) \\
& \quad \quad V2q))) \Rightarrow ((ap (ap c_2Earithmetic_2E_2A V0n) (ap (ap c_2Earithmetic_2EMOD \\
& \quad \quad V1p) V2q)) = (ap (ap c_2Earithmetic_2EMOD (ap (ap c_2Earithmetic_2E_2A \\
& \quad \quad V0n) V1p)) (ap (ap c_2Earithmetic_2E_2A V0n) V2q)))))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& (\forall V0b \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) \\
& V0b)) \Rightarrow (\forall V1n \in ty_2Enum_2Enum. (\forall V2m \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Eprim_rec_2E_3C (ap (ap c_2Earithmetic_2EEXP V0b) \\
& V2m)) (ap (ap c_2Earithmetic_2EEXP V0b) V1n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& V2m) V1n)))))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in ty_2Enum_2Enum. (\forall V1q \in ty_2Enum_2Enum. (\\
& (p (ap (ap c_2Eprim_rec_2E_3C (ap (ap c_2Earithmetic_2E_2B (ap \\
& (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 \\
& c_2Earithmetic_2EZERO))) V0p)) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) (ap (ap \\
& c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 \\
& c_2Earithmetic_2EZERO))) V1q))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL (ap \\
& c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) V0p)) (ap (ap \\
& c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 \\
& c_2Earithmetic_2EZERO))) V1q))))))
\end{aligned} \tag{34}$$

Assume the following.

$$True \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\
& (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2. ((\neg (\neg (p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{38}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (39)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (40)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (41)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in \\ & (2^{A_27a}).((\forall V2x \in A_27a.((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow \\ & ((\forall V3x \in A_27a.(p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A_27a.(p\ (\\ & ap\ V1Q\ V4x)))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\ & 2^{A_27a}).((p\ V0P) \wedge (\forall V2x \in A_27a.(p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in \\ & A_27a.((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x)))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (\\ & 2^{A_27a}).((\forall V2x \in A_27a.((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q))) \Leftrightarrow ((\forall V3x \in \\ & A_27a.(p\ (ap\ V1P\ V3x))) \vee (p\ V0Q)))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee (\\ & (p\ V1B) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C)))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V1B) \wedge \\ & (p\ V2C)) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A)))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (48)$$

Assume the following.

$$(\forall V0v \in 2.((p (ap c.2Ebool.2EBOUNDED V0v)) \Leftrightarrow True)) \quad (49)$$

Assume the following.

$$\begin{aligned} &(((ap c.2Enum.2ESUC c.2Earithmetic.2EZERO) = (ap c.2Earithmetic.2EBIT1 \\ &\quad c.2Earithmetic.2EZERO)) \wedge ((\forall V0n \in ty.2Enum.2Enum.((ap \\ &\quad c.2Enum.2ESUC (ap c.2Earithmetic.2EBIT1 V0n)) = (ap c.2Earithmetic.2EBIT2 \\ &\quad V0n))) \wedge (\forall V1n \in ty.2Enum.2Enum.((ap c.2Enum.2ESUC (ap c.2Earithmetic.2EBIT2 \\ &\quad V1n)) = (ap c.2Earithmetic.2EBIT1 (ap c.2Enum.2ESUC V1n)))))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned} &(\forall V0n \in ty.2Enum.2Enum.(p (ap (ap c.2Eprim_rec.2E.3C c.2Enum.2E0) \\ &\quad (ap c.2Enum.2ESUC V0n)))) \end{aligned} \quad (51)$$

Assume the following.

$$\begin{aligned} &(\forall V0n \in ty.2Enum.2Enum.(p (ap (ap c.2Eprim_rec.2E.3C V0n) \\ &\quad (ap c.2Enum.2ESUC V0n)))) \end{aligned} \quad (52)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (53)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (54)$$

Assume the following.

$$\begin{aligned} &(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\ &\quad ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned} &(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\ &\quad ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \end{aligned} \quad (56)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (57)$$

Assume the following.

$$\begin{aligned} &(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ &\quad (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\ &\quad p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ &\quad ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow (\\
& (p \vee V1q) \wedge (p \vee V2r))) \Leftrightarrow (((p \vee V0p) \vee (\neg(p \vee V1q)) \vee \neg(p \vee V2r))) \wedge (((p \vee V1q) \vee \\
& (\neg(p \vee V0p))) \wedge ((p \vee V2r) \vee \neg(p \vee V0p))))))))) \\
& \hspace{15em} (59)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow (\\
& (p \vee V1q) \vee (p \vee V2r))) \Leftrightarrow (((p \vee V0p) \vee \neg(p \vee V1q)) \wedge ((p \vee V0p) \vee \neg(p \vee V2r))) \wedge \\
& ((p \vee V1q) \vee ((p \vee V2r) \vee \neg(p \vee V0p))))))))) \\
& \hspace{15em} (60)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow (\\
& (p \vee V1q) \Rightarrow (p \vee V2r))) \Leftrightarrow (((p \vee V0p) \vee (p \vee V1q)) \wedge (((p \vee V0p) \vee \neg(p \vee V2r))) \wedge (\\
& \neg(p \vee V1q) \vee ((p \vee V2r) \vee \neg(p \vee V0p))))))))) \\
& \hspace{15em} (61)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \vee V0p) \Leftrightarrow \neg(p \vee V1q)) \Leftrightarrow (((p \vee V0p) \vee \\
& (p \vee V1q)) \wedge (\neg(p \vee V1q) \vee \neg(p \vee V0p)))))) \\
& \hspace{15em} (62)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \Rightarrow (p \vee V1q))) \Rightarrow (p \vee V0p)))) \\
& \hspace{15em} (63)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \Rightarrow (p \vee V1q))) \Rightarrow \neg(p \vee V1q)))) \\
& \hspace{15em} (64)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \vee (p \vee V1q))) \Rightarrow \neg(p \vee V0p)))) \\
& \hspace{15em} (65)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \vee (p \vee V1q))) \Rightarrow \neg(p \vee V1q)))) \\
& \hspace{15em} (66)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. ((\neg(\neg(p \vee V0p))) \Rightarrow (p \vee V0p))) \\
& \hspace{15em} (67)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow ((\forall V0n \in \text{ty_2Enum_2Enum}. \\
& ((\text{ap } (\text{ap } \text{c_2Enumeral_bit_2EiMOD_2EXP } \text{c_2Enum_2E0}) V0n) = \text{c_2Earithmetic_2EZERO})) \wedge \\
& ((\forall V1x \in \text{ty_2Enum_2Enum}. (\forall V2n \in A_{27a}. ((\text{ap } (\text{ap } \text{c_2Enumeral_bit_2EiMOD_2EXP} \\
& \quad V1x) \text{c_2Earithmetic_2EZERO}) = \text{c_2Earithmetic_2EZERO}))) \wedge ((\forall V3x \in \\
& \text{ty_2Enum_2Enum}. (\forall V4n \in \text{ty_2Enum_2Enum}. ((\text{ap } (\text{ap } \text{c_2Enumeral_bit_2EiMOD_2EXP} \\
& \quad (\text{ap } \text{c_2Earithmetic_2ENUMERAL } (\text{ap } \text{c_2Earithmetic_2EBIT1 } V3x))) \\
& \quad (\text{ap } \text{c_2Earithmetic_2EBIT1 } V4n)) = (\text{ap } \text{c_2Earithmetic_2EBIT1 } (\\
& \quad \text{ap } (\text{ap } \text{c_2Enumeral_bit_2EiMOD_2EXP } (\text{ap } (\text{ap } \text{c_2Earithmetic_2E_2D} \\
& \quad (\text{ap } \text{c_2Earithmetic_2ENUMERAL } (\text{ap } \text{c_2Earithmetic_2EBIT1 } V3x))) \\
& \quad (\text{ap } \text{c_2Earithmetic_2ENUMERAL } (\text{ap } \text{c_2Earithmetic_2EBIT1 } \text{c_2Earithmetic_2EZERO})))))) \\
& \quad V4n)))))) \wedge ((\forall V5x \in \text{ty_2Enum_2Enum}. (\forall V6n \in \text{ty_2Enum_2Enum}. \\
& ((\text{ap } (\text{ap } \text{c_2Enumeral_bit_2EiMOD_2EXP } (\text{ap } \text{c_2Earithmetic_2ENUMERAL} \\
& \quad (\text{ap } \text{c_2Earithmetic_2EBIT2 } V5x))) (\text{ap } \text{c_2Earithmetic_2EBIT1 } V6n)) = \\
& \quad (\text{ap } \text{c_2Earithmetic_2EBIT1 } (\text{ap } (\text{ap } \text{c_2Enumeral_bit_2EiMOD_2EXP} \\
& \quad (\text{ap } \text{c_2Earithmetic_2ENUMERAL } (\text{ap } \text{c_2Earithmetic_2EBIT1 } V5x))) \\
& \quad V6n)))))) \wedge ((\forall V7x \in \text{ty_2Enum_2Enum}. (\forall V8n \in \text{ty_2Enum_2Enum}. \\
& ((\text{ap } (\text{ap } \text{c_2Enumeral_bit_2EiMOD_2EXP } (\text{ap } \text{c_2Earithmetic_2ENUMERAL} \\
& \quad (\text{ap } \text{c_2Earithmetic_2EBIT1 } V7x))) (\text{ap } \text{c_2Earithmetic_2EBIT2 } V8n)) = \\
& \quad (\text{ap } \text{c_2Enumeral_2EiDUB } (\text{ap } (\text{ap } \text{c_2Enumeral_bit_2EiMOD_2EXP} \\
& \quad (\text{ap } (\text{ap } \text{c_2Earithmetic_2E_2D } (\text{ap } \text{c_2Earithmetic_2ENUMERAL } (\text{ap} \\
& \quad \text{c_2Earithmetic_2EBIT1 } V7x))) (\text{ap } \text{c_2Earithmetic_2ENUMERAL } (\\
& \quad \text{ap } \text{c_2Earithmetic_2EBIT1 } \text{c_2Earithmetic_2EZERO})))))) (\text{ap } \text{c_2Enum_2ESUC} \\
& \quad V8n)))))) \wedge ((\forall V9x \in \text{ty_2Enum_2Enum}. (\forall V10n \in \text{ty_2Enum_2Enum}. \\
& ((\text{ap } (\text{ap } \text{c_2Enumeral_bit_2EiMOD_2EXP } (\text{ap } \text{c_2Earithmetic_2ENUMERAL} \\
& \quad (\text{ap } \text{c_2Earithmetic_2EBIT2 } V9x))) (\text{ap } \text{c_2Earithmetic_2EBIT2 } V10n)) = \\
& \quad (\text{ap } \text{c_2Enumeral_2EiDUB } (\text{ap } (\text{ap } \text{c_2Enumeral_bit_2EiMOD_2EXP} \\
& \quad (\text{ap } \text{c_2Earithmetic_2ENUMERAL } (\text{ap } \text{c_2Earithmetic_2EBIT1 } V9x))) \\
& \quad (\text{ap } \text{c_2Enum_2ESUC } V10n)))))))))
\end{aligned}$$