thm_2Enumpair_2Enapp__thm (TMW-bVK6yq77PKwhwmjVyaoJ1UsF3bpAYrGU)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \quad (1)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (2)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (3)$$

Definition 5 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 6 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 8 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (4)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \quad (5)$$

Definition 9 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

Definition 10 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 11 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Enumpair\_2Etri : \iota$ be given. Assume the following.

$$c\_2Enumpair\_2Etri \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}) \tag{7}$$

Definition 12 *We define* c_2Enumpair_2Enpair *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 13 *We define* c_2Enumpair_2Encons *to be* $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1t \in ty\_2Enum\_2En$

Let $c\_2Enumpair\_2Enlistrec : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Enumpair\_2Enlistrec\ A\_27a \in (((A\_27a^{ty\_2Enum\_2Enum})^{(((A\_27a^{A\_27a})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})})^{A\_27a}) \tag{8}$$

Definition 14 *We define* c_2Enumpair_2Enapp *to be* $\lambda V0l1 \in ty\_2Enum\_2Enum.\lambda V1l2 \in ty\_2Enum\_2En$

Assume the following.

$$True \tag{9}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{10}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{11}$$

Assume the following.

$$(\forall V0x \in ty\_2Enum\_2Enum.(\forall V1y \in ty\_2Enum\_2Enum.(\forall V2h \in ty\_2Enum\_2Enum.(\forall V3t \in ty\_2Enum\_2Enum.(((ap\ (ap\ \mathsf{c\_2Enumpair\_2Encons}\ V0x)\ V1y) = (ap\ (ap\ \mathsf{c\_2Enumpair\_2Encons}\ V2h)\ V3t)) \Leftrightarrow ((V0x = V2h) \wedge (V1y = V3t))))))) \tag{12}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0n \in A\_27a.(\forall V1f \in (((A\_27a^{A\_27a})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}).(\forall V2h \in ty\_2Enum\_2Enum.(\forall V3t \in ty\_2Enum\_2Enum.(((ap\ (ap\ (ap\ (c\_2Enumpair\_2Enlistrec\ A\_27a)\ V0n)\ V1f)\ \mathsf{c\_2Enum\_2E0}) = V0n) \wedge ((ap\ (ap\ (ap\ (c\_2Enumpair\_2Enlistrec\ A\_27a)\ V0n)\ V1f)\ (ap\ (ap\ \mathsf{c\_2Enumpair\_2Encons}\ V2h)\ V3t)) = (ap\ (ap\ (ap\ V1f\ V2h)\ V3t)\ (ap\ (ap\ (ap\ (c\_2Enumpair\_2Enlistrec\ A\_27a)\ V0n)\ V1f)\ V3t))))))))) \tag{13}$$

Theorem 1

$$(\forall V0nlist \in ty\_2Enum\_2Enum.(\forall V1h \in ty\_2Enum\_2Enum.(\forall V2t \in ty\_2Enum\_2Enum.(((ap\ (ap\ \mathsf{c\_2Enumpair\_2Enapp}\ \mathsf{c\_2Enum\_2E0})\ V0nlist) = V0nlist) \wedge ((ap\ (ap\ \mathsf{c\_2Enumpair\_2Enapp}\ (ap\ (ap\ \mathsf{c\_2Enumpair\_2Encons}\ V1h)\ V2t))\ V0nlist) = (ap\ (ap\ \mathsf{c\_2Enumpair\_2Encons}\ V1h)\ (ap\ (ap\ \mathsf{c\_2Enumpair\_2Enapp}\ V2t)\ V0nlist)))))))$$