

thm\_2Enumpair\_2Encons\_11  
 (TMGgoiANZfTFdYsmetrFX2rtu2SWcKRQxjs)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p \ P \Rightarrow p \ Q)$  of type  $\iota$ .

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})) (\lambda V1t1 \in 2. (ap (c\_2Ebool\_2E\_21 2) (\lambda V2t2 \in 2. inj\_o (V1t1 = V2t2)))))))$

**Definition 5** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2. inj\_o (V1t2 = V2t))))))$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (1)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty \ ty\_2Enum\_2Enum \quad (2)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (3)$$

**Definition 6** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

**Definition 7** We define  $c\_2Earithmetic\_2ZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (4)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (5)$$

**Definition 8** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ V0m)$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$

**Definition 9** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n))$

**Definition 10** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Enumpair\_2Etri : \iota$  be given. Assume the following.

$$c\_2Enumpair\_2Etri \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}) \quad (7)$$

**Definition 11** We define  $c\_2Enumpair\_2Enpair$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 12** We define  $c\_2Enumpair\_2Encons$  to be  $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1t \in ty\_2Enum\_2Enum.$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m) \\ & V2p) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n) V2p)) \Leftrightarrow (V0m = V1n)))))) \end{aligned} \quad (8)$$

Assume the following.

$$True \quad (9)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (10)$$

Assume the following.

$$\begin{aligned} & (\forall V0x1 \in ty\_2Enum\_2Enum.(\forall V1y1 \in ty\_2Enum\_2Enum.(\forall V2x2 \in ty\_2Enum\_2Enum.(\forall V3y2 \in ty\_2Enum\_2Enum.(( \\ & (((ap\ (ap\ c\_2Enumpair\_2Enpair\ V0x1) V1y1) = (ap\ (ap\ c\_2Enumpair\_2Enpair\ V2x2) V3y2)) \Leftrightarrow ((V0x1 = V2x2) \wedge (V1y1 = V3y2))))))) \end{aligned} \quad (11)$$

**Theorem 1**

$$\begin{aligned} & (\forall V0x \in ty\_2Enum\_2Enum.(\forall V1y \in ty\_2Enum\_2Enum.(\forall V2h \in ty\_2Enum\_2Enum.(\forall V3t \in ty\_2Enum\_2Enum.(( \\ & (ap\ (ap\ c\_2Enumpair\_2Encons\ V0x) V1y) = (ap\ (ap\ c\_2Enumpair\_2Encons\ V2h) V3t)) \Leftrightarrow ((V0x = V2h) \wedge (V1y = V3t))))))) \end{aligned}$$