thm_2Enumpair_2Enlen__thm (TMFtuf4PsXEmDkTsHyGiGRbebTuwYR3naXV)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 3 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 4 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 6 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

Definition 7 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 8 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Enumpair\_2Etri : \iota$ be given. Assume the following.

$$c\_2Enumpair\_2Etri \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}) \tag{7}$$

Definition 9 *We define* c_2Enumpair_2Enpair *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Definition 10 *We define* c_2Enumpair_2Encons *to be* $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1t \in ty\_2Enum\_2Enu$

Definition 11 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 12 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $c\_2Enumpair\_2Enlistrec : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Enumpair\_2Enlistrec\ A\_27a \in (((A\_27a^{ty\_2Enum\_2Enum})^{(((A\_27a^{A\_27a})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})})^{A\_27a}) \tag{8}$$

Definition 13 *We define* c_2Enumpair_2Enlen *to be* $(ap\ (ap\ (c\_2Enumpair\_2Enlistrec\ ty\_2Enum\_2Enum$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V2p) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V2p)) \Leftrightarrow (V0m = V1n))))) \tag{9}$$

Assume the following.

$$True \tag{10}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{11}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0n \in A\_27a.(\forall V1f \in (((A\_27a^{A\_27a})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}).(\forall V2h \in ty\_2Enum\_2Enum.(\forall V3t \in ty\_2Enum\_2Enum.(((ap\ (ap\ (ap\ (c\_2Enumpair\_2Enlistrec\ A\_27a)\ V0n)\ V1f)\ \mathsf{c\_2Enum\_2E0}) = V0n) \wedge ((ap\ (ap\ (ap\ (c\_2Enumpair\_2Enlistrec\ A\_27a)\ V0n)\ V1f)\ (ap\ (ap\ \mathsf{c\_2Enumpair\_2Encons}\ V2h)\ V3t)) = (ap\ (ap\ (ap\ V1f\ V2h)\ V3t)\ (ap\ (ap\ (ap\ (c\_2Enumpair\_2Enlistrec\ A\_27a)\ V0n)\ V1f)\ V3t)))))))) \tag{12}$$

Theorem 1

$$(\forall V0h \in ty\_2Enum\_2Enum.(\forall V1t \in ty\_2Enum\_2Enum.(((ap\ \mathsf{c\_2Enumpair\_2Enlen}\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0}) \wedge ((ap\ \mathsf{c\_2Enumpair\_2Enlen}\ (ap\ (ap\ \mathsf{c\_2Enumpair\_2Encons}\ V0h)\ V1t)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Enumpair\_2Enlen}\ V1t))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))))$$