

thm\_2Enumpair\_2Enlist\_\_ind  
(TMUzmxjpRH5CHFbTWQPzTrVSTnZJYvAGiQg)

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**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.$ if  $(\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p$   
of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$   
of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A$

**Definition 4** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$   
of type  $\iota$ .

**Definition 5** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V$

**Definition 6** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}$

**Definition 7** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2$

**Definition 8** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (omega^{omega}) \tag{3}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{4}$$

**Definition 9** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num ($

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{5}$$

**Definition 10** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{6}$$

Let  $c\_2Enumpair\_2Einvtri0 : \iota$  be given. Assume the following.

$$c\_2Enumpair\_2Einvtri0 \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \tag{7}$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c\_2Epair\_2ESND\ A.27a\ A.27b \in (A.27b)^{(ty\_2Epair\_2Eprod\ A.27a\ A.27b)} \tag{8}$$

**Definition 11** We define  $c\_2Enumpair\_2Einvtri$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (c\_2Epair\_2ESND\ ty\_2Enum\_2Enum\ V0n))$ .

Let  $c\_2Enumpair\_2Etri : \iota$  be given. Assume the following.

$$c\_2Enumpair\_2Etri \in (ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum} \tag{9}$$

Let  $c\_2Earithmetic\_2E.2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E.2B \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \tag{10}$$

Let  $c\_2Earithmetic\_2E.2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E.2D \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \tag{11}$$

**Definition 12** We define  $c\_2Enumpair\_2Enfst$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E.2B\ V0n))$ .

**Definition 13** We define  $c\_2Enumpair\_2Enpair$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E.2D\ V0m\ V1n))$ .

**Definition 14** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

**Definition 15** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E.2B\ V0n))$ .

**Definition 16** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 17** We define  $c\_2Enumpair\_2Encons$  to be  $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1t \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E.2B\ V0h\ V1t))$ .

**Definition 18** We define  $c\_2Enumpair\_2Ensnd$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E.2D\ V0n\ V0n))$ .

**Definition 19** We define  $c\_Ebool\_2E\_EF$  to be  $(ap (c\_Ebool\_2E\_21\ 2) (\lambda V0t \in 2.V0t))$ .

**Definition 20** We define  $c\_Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_Emin\_2E\_3D\_3D\_3E\ V0t) c\_Ebool\_2E\_21\ 2))$ .

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((V0m = c\_2Enum\_2E0) \vee (\exists V1n \in ty\_2Enum\_2Enum.(V0m = (ap c\_2Enum\_2ESUC\ V1n)))))) \quad (12)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap c\_2Enum\_2ESUC\ V0m) = (ap (ap c\_2Earithmetic\_2E\_2B\ V0m) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))))) \quad (13)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D (ap c\_2Enum\_2ESUC\ V0m)) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))) = V0m)) \quad (14)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.(((ap (ap c\_2Earithmetic\_2E\_2B\ V0m) V2p) = (ap (ap c\_2Earithmetic\_2E\_2B\ V1n) V2p)) \Leftrightarrow (V0m = V1n)))))) \quad (15)$$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (18)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t) \Leftrightarrow (p\ V1x)))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \quad (20)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (21)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (22)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (24)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))) \quad (25)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.(((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27))))) \quad (26)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg((ap \ c\_2Enum\_2ESUC \ V0n) = c\_2Enum\_2E0))) \quad (27)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap \ (ap \ c\_2Enumpair\_2Enpair \ (ap \ c\_2Enumpair\_2Enfst \ V0n)) \ (ap \ c\_2Enumpair\_2Ensnd \ V0n)) = V0n)) \quad (28)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in (((2ty\_2Enum\_2Enum)((A\_27a^{A\_27a})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}))_{A\_27a} \\ ((\forall V1n \in A\_27a.(\forall V2f \in (((A\_27a^{A\_27a})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}). \\ (\forall V3l \in ty\_2Enum\_2Enum.(((\neg(V3l = c\_2Enum\_2E0)) \Rightarrow (p \ (ap \\ (ap \ (ap \ V0P \ V1n) \ V2f) \ (ap \ c\_2Enumpair\_2Ensnd \ (ap \ (ap \ c\_2Earithmetic\_2E\_2D \\ V3l) \ (ap \ c\_2Earithmetic\_2ENUMERAL \ (ap \ c\_2Earithmetic\_2EBIT1 \\ c\_2Earithmetic\_2EZERO)))))) \Rightarrow (p \ (ap \ (ap \ (ap \ V0P \ V1n) \ V2f) \ V3l)))))) \Rightarrow \\ (\forall V4v \in A\_27a.(\forall V5v1 \in (((A\_27a^{A\_27a})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}). \\ (\forall V6v2 \in ty\_2Enum\_2Enum.(p \ (ap \ (ap \ (ap \ V0P \ V4v) \ V5v1) \ V6v2)))))) \quad (29) \end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (32)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (33)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (34)$$

**Theorem 1**

$$\begin{aligned} & (\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p (ap V0P c\_2Enum\_2E0)) \wedge \\ & (\forall V1h \in ty\_2Enum\_2Enum.(\forall V2t \in ty\_2Enum\_2Enum.( \\ & (p (ap V0P V2t)) \Rightarrow (p (ap V0P (ap (ap c\_2Enumpair\_2Encons V1h) V2t)))))) \Rightarrow \\ & (\forall V3n \in ty\_2Enum\_2Enum.(p (ap V0P V3n)))))) \end{aligned}$$