

thm_2Enumpair_2Enmap__thm (TMNzqbcW- erMtyWeG1mstsgedf59ccwWLuUnV)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 4 We define `c_2Ebool_2E_2F` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2.V2t))))$

Let `c_2Enum_2EZERO__REP` : ι be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \text{omega} \tag{1}$$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty } ty_2Enum_2Enum \tag{2}$$

Let `c_2Enum_2EABS__num` : ι be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\text{omega}}) \tag{3}$$

Definition 7 We define `c_2Enum_2E0` to be $(\text{ap } c_2Enum_2EABS_num \ c_2Enum_2EZERO_REP)$.

Definition 8 We define `c_2Earithmic_2EZERO` to be `c_2Enum_2E0`.

Let `c_2Enum_2EREP__num` : ι be given. Assume the following.

$$c_2Enum_2EREP_num \in (\text{omega}^{ty_2Enum_2Enum}) \tag{4}$$

Let `c_2Enum_2ESUC__REP` : ι be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\text{omega}^{\text{omega}}) \tag{5}$$

Definition 9 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 10 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 11 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Enumpair_2Etri : \iota$ be given. Assume the following.

$$c_2Enumpair_2Etri \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}) \quad (7)$$

Definition 12 We define $c_2Enumpair_2Enpair$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 13 We define $c_2Enumpair_2Encons$ to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1t \in ty_2Enum_2Enum$

Let $c_2Enumpair_2Enlistrec : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Enumpair_2Enlistrec\ A.27a \in \\ (((A.27a^{ty_2Enum_2Enum})^{((A.27a^{A.27a})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})})^{A.27a}) \quad (8)$$

Definition 14 We define $c_2Enumpair_2Enmap$ to be $\lambda V0f \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}).(ap\ (ap$

Assume the following.

$$True \quad (9)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (10)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow \\ True)) \quad (11)$$

Assume the following.

$$(\forall V0x \in ty_2Enum_2Enum.(\forall V1y \in ty_2Enum_2Enum.(\\ \forall V2h \in ty_2Enum_2Enum.(\forall V3t \in ty_2Enum_2Enum.(\\ (ap\ (ap\ c_2Enumpair_2Encons\ V0x)\ V1y) = (ap\ (ap\ c_2Enumpair_2Encons \\ V2h)\ V3t)) \Leftrightarrow ((V0x = V2h) \wedge (V1y = V3t)))))) \quad (12)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0n \in A.27a. (\forall V1f \in \\
& \quad (((A.27a^{A.27a})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}). (\forall V2h \in \\
& \quad ty_2Enum_2Enum. (\forall V3t \in ty_2Enum_2Enum. (((ap\ (ap\ (ap\ (c_2Enumpair_2Enlistrec \\
& \quad A.27a)\ V0n)\ V1f)\ c_2Enum_2E0) = V0n) \wedge ((ap\ (ap\ (ap\ (c_2Enumpair_2Enlistrec \\
& \quad A.27a)\ V0n)\ V1f)\ (ap\ (ap\ c_2Enumpair_2Encons\ V2h)\ V3t)) = (ap\ (ap \\
& \quad (ap\ V1f\ V2h)\ V3t)\ (ap\ (ap\ (ap\ (c_2Enumpair_2Enlistrec\ A.27a)\ V0n) \\
& \quad V1f)\ V3t)))))))))
\end{aligned} \tag{13}$$

Theorem 1

$$\begin{aligned}
& (\forall V0f \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}). (\forall V1h \in \\
& \quad ty_2Enum_2Enum. (\forall V2t \in ty_2Enum_2Enum. (((ap\ (ap\ c_2Enumpair_2Enmap \\
& \quad V0f)\ c_2Enum_2E0) = c_2Enum_2E0) \wedge ((ap\ (ap\ c_2Enumpair_2Enmap \\
& \quad V0f)\ (ap\ (ap\ c_2Enumpair_2Encons\ V1h)\ V2t)) = (ap\ (ap\ c_2Enumpair_2Encons \\
& \quad (ap\ V0f\ V1h))\ (ap\ (ap\ c_2Enumpair_2Enmap\ V0f)\ V2t)))))))))
\end{aligned}$$